

Analysis of Estimation Error of Skewness and Kurtosis of Bunimovich-Rice Processes with Exponentially Power Waveform of Pulses

A. I. Krasil'nikov^{1*} and V. S. Beregun^{2**}

¹*Institute of Engineering Thermophysics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine*

²*National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine*

*ORCID: [0000-0001-5666-6459](https://orcid.org/0000-0001-5666-6459), e-mail: tangorov@ukr.net

**ORCID: [0000-0002-6673-4491](https://orcid.org/0000-0002-6673-4491), e-mail: viktorberegun@i.ua

Received February 4, 2020

Revised August 12, 2020

Accepted August 20, 2020

Abstract—Mathematical expectations and variances of estimates of skewness and kurtosis coefficients of the noise signal model have been derived in this study. The specified noise signals represent the Bunimovich–Rice processes with exponentially power waveform of pulses expressed in terms of cumulant coefficients of the specified processes. It is shown that the distribution of instantaneous values of Bunimovich–Rice processes is quite different from the Gaussian distribution. The root-mean-square and relative estimation errors of skewness and kurtosis coefficients depending on of the time constant and waveform parameter of elementary pulses, as well as distributions of pulse amplitudes (degenerate and gamma distributions) and their intensities are analyzed. Expressions for finding the minimal sample volumes are obtained that ensure the specified values of relative errors in estimating the skewness and kurtosis coefficients of Bunimovich–Rice processes. The minimal sample volumes depending on parameters of these processes have been determined that ensure the relative estimation errors, which do not exceed 1%.

DOI: 10.3103/S0735272720080051

1. INTRODUCTION

Noise signals, such as noises of electronic appliances and systems [1]–[4], and also the acoustic [5]–[8] and biomedical [9]–[11] noise signals, are sources of information about fluctuation phenomena that occur in the process of natural functioning of many physical and technical entities under investigation.

In particular, the 1/f-noise (flicker noise) [1]–[4] occurring in almost all semiconductor devices (photoresistors, diodes, field-effect and bipolar transistors, etc.), in metal films and others, contain information about the hidden defects of different electronic devices and systems.

Noise signals and their probabilistic characteristics are the subject of studies in statistical radio engineering [12]–[15]. They are widely used in different applications of radioelectronics, in particular, in solid-state electronics and integrated circuitry for quality control and electronic device diagnostics [1], [2], in radioelectronic devices and systems for medical noise diagnostics [9], [10], etc.

The efficiency of further application of noise signals for investigating the properties of physical and technical entities essentially depends on the choice of mathematical model of these signals and determination of the informativity of probabilistic characteristics based on this model.

The Bunimovich–Rice processes [12], [13] are the simplest and most common model of noise signals representing the result of summation of a large number of elementary pulses with random parameters:

$$\xi(t) = \sum_{k=1}^{v(t)} \eta_k h(t - t_k), \quad (1)$$

where the Poisson homogeneous process $v(t)$ with intensity $\lambda > 0$ describes the number of pulses on interval $[0; t]$; time instants of pulse originations t_k are a homogeneous Poisson flow of events; nonrandom function

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

ADDITIONAL INFORMATION

The initial version of this paper in Russian is published in the journal "Izvestiya Vysshikh Uchebnykh Zavedenii. Radioelektronika," ISSN 2307-6011 (Online), ISSN 0021-3470 (Print) on the link <http://radio.kpi.ua/article/view/S0021347020080051> with DOI: [10.20535/S0021347020080051](https://doi.org/10.20535/S0021347020080051).

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