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Discrete LS Estimates of Correlation Function of Bi-Periodically Correlated Random Signals

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Abstract—Analysis of discrete estimates of bi-periodically correlated random processes (BPCRP)—mathematical models of signals with double stochastic periodicity was performed using least squares (LS) technique. It is shown that LS utilization allows to avoid systematic errors related to leakage effect. Expressions for estimate bias and dispersion were obtained, allowing determination systematic and mean square errors depending on discretization step, sample number and signal parameters were obtained. For quadrature BPCRP model discrete and continuous dispersions of LS estimation of correlation components were compared. Recommendations for discretization step are given.

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INTRODUCTION

Realization length and discretization step are one of the key parameters which determine errors for statistical processing of experimental data. For a reasonable selection of these values it is needed to consider a goal of the statistical processing, which is performed using a technique based on a respective mathematical model of the signal.

If a signal is investigated using observation data can be described by stationary random process, and the goal of analysis is to find estimates of the correlation function and power spectral density, realization length and discretization step should be chosen according to required systematic and mean square errors of these values. Analysis results for dependencies of such errors for processing and signals parameters are known [1–3].

Significant progress in this field has been made in recent years in correlation and spectral analysis of periodically correlated random processes (PCRP) [4–11]. PCRP are an adequate model of oscillations with inherent traits of interacting periodicity and stochasticity. To this type of oscillations belong next signals: information carriers in radio communications, telemetry, radiolocation, noise in atmosphere and ocean, noise in electronic circuits, etc. [4, 7, 8]. Stochastic periodicity is acquired as a result of modulation, discretization, scanning, coding etc. Solution of signals conversion, processing, filtration, diagnostics problems using PCRP-model gives an opportunity to substantially improve obtained results [8–13].

However, during analysis of artificial and inartificial signals it is often a case when stochastic periodicity of one period interacts with stochastic periodicity of another. For example, in communications signals appear periodicities induced by carrier periodicity and rhythmic changes in modulating signal [8, 11, 13], discretization, coding [4, 6, 10]. In power supply systems it is daily, weekly, yearly periodicities [8]. In electric machines vibration signals polyrhythmic variability is caused by different velocities of various units [8, 9, 12]. Double rhythmic is characteristic to medical equipment signals [7]. These signals properties should be taken into account while designing radio electronic devices for conversion and processing of such signals for determining parameters of systems in which they appear. This is a basis for successful solution of non-destructive testing problem [14], diagnostics, including medical [7] and vibration [15].

Probabilistic model of a double rhythmic is a bi-periodically correlated random processes (BPCRP) [8, 16]. In the basis of such model composition it is an assumption that an interaction of different period

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

ADDITIONAL INFORMATION

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