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Estimation of Optimal Parameter of Regularization of Signal Recovery[1]

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Abstract—In this paper there are researched regularizing properties of discretization in a space of output signals for some linear operator equation with noisy data. The essence of proposed method is selection of discretization level which is a parameter of the regularization in this context by the principle of equality of random and deterministic components of the input signal recovering error. It is shown the method, i.e. the solution which is discrete by input signal is stable to small inaccuracies in input signal. At that in case of definite level of output signal measurements inaccuracy the recovering error of input signal is unambiguously defined by input signal sampling increment that allows to select reasonably the regularization parameter for specific criterion, for example, for definite measurements inaccuracy. Specific calculations and examples are represented in explicit form for single-dimension case but this does not restricts generality of proposed method.

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1. INTRODUCTION

Mathematic model of stationery measurement systems is integral equation of the first genus with differential kernel [1, 2]

$$\int g(\rho)h(r-\rho)\mathrm{d}\rho = f(r), \tag{1}$$

where $h(r-\rho)$ is pulse response of the system (point dissipation function).

There are known appearing principal and practical difficulties of analysis of (1). Transformation kernel $h(r-\rho)$ in (1) reflects smoothing properties of the device or measurement method as a whole. For existing measurement system there is a problem of obtain of the correspondence of output signal $f(r)$ and desired signal $g(\rho)$ that requires the solution of integral equation (1).

From mathematic viewpoint it is necessary to obtain the solution of integral equation of the first genus (1) that is ill-posed problem [1-3] regarding definition that this problem solution is essentially sensitive to small data modification (i.e. function $f(r)$).

Thus the specificity of the recovering process for input signal is incorrectness (1) following from presence of right part of the equation and sensitivity of the problem solution to the signal $f(r)$ behavior [1].

The reason of absence of continuous dependence of the solution on the equation right part is fact that for all physically realizable systems their frequency spectrum of pulse response decreases in area of high spatial frequencies. Therefore in case of inverse transformation from output to the input signal high frequency components with small amplitude are amplified resulting in destroying of the information about input signal.

We note inspite of ill-posed problems are of interest during more than last 50 years such problems and their regularization methods are actual till now [4] applying in field of heat conductance [5, 6], medicine [7, 8] and others. New applications include development of fiber optic communication based on the methods signal processing by means of inverse disperse problem (non-linear Fourier transformation) [9]. Last time

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