

# Sub-Nyquist Sampling and Parameters Estimation of Wideband LFM Signals Based on FRFT

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Received in final form July 22, 2018

**Abstract**—Last years, most sub-Nyquist sampling and parameters estimation methods for linear frequency modulated (LFM) signals are based on compressed sensing (CS) theory. However, nearly all CS reconstruction algorithms are with high computational complexity and difficult to be implemented in hardware. In this paper, a novel framework of sub-Nyquist sampling and low-complexity parameters estimation for LFM signals is proposed. The incoherent sampling in CS theory is introduced into the construction of sub-Nyquist sampling system, but no CS reconstruction algorithm is employed in the estimation of parameters. Based on the energy aggregation of LFM signals in the proper fractional Fourier transform (FRFT) domain, the chirp rate and center frequency can be estimated by linear operations. Accordingly, the proposed estimation method is easily realized compared with existing estimation methods based on CS. Simulation results verify its effectiveness and accuracy.

DOI: 10.3103/S0735272718080010

## 1. INTRODUCTION

Linear frequency modulated (LFM) signals, also called “chirp” signals, are widely applied in various areas, such as communication, sonar, and radar. As LFM signals in many applications are wideband signals with a large time–bandwidth product, the wide bandwidth brings high pressure to the signal acquisition and processing.

According to compressed sensing (CS) theory [1, 2], when a signal has a sparse representation in some basis, a relatively few non-adaptive random linear measurements can capture all the information necessary to faithfully reconstruct the signal. In some situations where a signal reconstruction is unnecessary, the CS theory can be utilized to estimate signal parameters by directly processing the CS data [3–5].

Several parameter estimation methods for LFM signals based on CS theory have been proposed. In [6–8] the main idea is to design a redundant signal-matched dictionary, and then find the best matched element of the signal in the dictionary. Parameters of the LFM signal can be estimated by the position of this element. As the super-resolution estimation of parameters greatly increases the number of elements in the dictionary, a high computational complexity is required to get accurate estimates of signal parameters.

In [9], based on the energy concentration property of LFM signals in the proper order fractional Fourier transform (FRFT) domain, an FRFT based sparsity basis is adopted for CS to estimate signal parameters. The scale of the discrete FRFT (DFRFT) dictionary is much smaller than that of the signal-matched dictionary. However, if there is no a prior information about the proper order, several DFRFT dictionaries with different orders need to be constructed. And to find out the best order, all the sparse representations corresponding to the dictionaries need to be recovered using CS reconstruction algorithms, that requires a huge amount of computation.

In this paper a two-channel sub-Nyquist sampling system is employed to sample wideband LFM signals, and a novel parameter estimation method is proposed based on the energy aggregation of LFM signals in the proper FRFT domain. The employed two-channel sampling system consists of an undersampling channel and a random demodulation (RD) [10] channel. Then, the estimate of chirp rate, as well as information related to the center frequency, can be obtained by a two-dimensional search in DFRFT plane of the undersampled sequence. Combined with the relevant information, the center frequency can be estimated from the output of RD by linear operations. Consequently, the proposed estimation method has a low computational complexity due to the absence of CS reconstruction algorithms.

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