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Odd Symmetry of Weights Vector of Symmetrical Antenna Arrays with Linear Constraints

V. I. Djigan

National Research University of Electronic Technology, Moscow, Russia
ORCID: 0000-0001-7485-1623, e-mail: djigan@yandex.ru

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Abstract—The paper provides proof of the odd symmetry of the vector of weight coefficients obtained on the basis of the least squares criterion in the linear antenna array with linear constraints and desired signal. Pairs of symmetrical elements of such vector are complex-conjugate to one another. For ensuring this property, the vector of constrained parameters (array pattern values in the directions of interest) must be real-valued, but need not be symmetrical. The odd symmetry of the antenna array vectors of input signals and weights makes it possible for such array to develop adaptive algorithms based on real-valued arithmetic. In this case, such algorithms have the number of arithmetic operations per iteration two or four times less as compared to the equivalent number of real-valued arithmetic operations of similar algorithms in the complex-valued arithmetic. The paper presents the results of comparative simulation of algorithms in the complex-valued and real-valued arithmetic. These results indicate that an adaptive algorithm using the real-valued arithmetic ensures (1.5–2)-fold shorter transient and deeper (by 2–3 dB) valleys in the steady state of the array pattern in the directions of sources of adaptively suppressed interferences as compared to the algorithm using the complex-valued arithmetic.

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INTRODUCTION

Adaptive signal processing is one of the key technologies used in equipment of modern systems of communications, radiolocation and radio navigation, and also household appliances. One of the elements of such equipment is a directional antenna, the implementation of which in the form of antenna array makes it possible without employing mechanical means to shift the main lobe of radiation pattern (RP), increase the signal-to-thermal noise ratio in the array output signal, and also to reduce the interference level from external sources in this signal at the expense of RP shape variation.

Antenna arrays possessing the last property are called adaptive (AAA) [1]. The suppression of interferences in such antennas is provided at the expense of valleys formed in RP in the interference source directions unknown at the receiving side. The variation of RP shape is achieved at the expense of changing the values of weight coefficients that are used for signal weighting in each of AAA channels before the summation for forming its output signal.

The weight coefficients are calculated using different adaptive algorithms [2–5] by processing the AAA input and output signals and using additional information regarding the source of useful signal. In radio communication systems, where the direction of received signal source is usually known, such information represents the direction employed in linearly constrained (LC) adaptive algorithms [6–9].

The algorithms of signal processing in AAA make use of the complex-valued arithmetic, because the amplitudes and phases define the signals received by antenna arrays. If AAA is symmetric, i.e. its phase center coincides with the geometric one, the number of arithmetic operations in specific adaptive algorithms can be reduced by half at the expense of real-valued arithmetic used for most calculations in these algorithms. Such array is called odd-symmetric, hereafter referred to as symmetric, because the pairs of its weight coefficients for signals received by antennas symmetrically arranged in aperture are complex-conjugate.

Estimates of the correlation matrix and the cross-correlation vector of signals in symmetric AAA using the real-valued arithmetic and based on the direct-reverse averaging also ensure a shorter transient and on average larger (by several decibels) interference suppression in steady state mode than in AAA using the complex-valued arithmetic. Papers [10–12] form the mathematical basis for signal processing in AAA with odd symmetry, while their results are used in a number of papers [13–23].

REFERENCES

1. R. T. Compton, *Adaptive Antennas: Concepts and Performance* (Prentice Hall, 1988).
2. P. S. R. Diniz, *Adaptive Filtering. Algorithms and Practical Implementation*, 4th ed. (Springer, 2013). DOI: 10.1007/978-1-4614-4106-9.
3. B. Farhang-Boroujeny, *Adaptive Filters: Theory and Applications*, 2nd ed. (John Willey and Sons, 2013).
4. S. O. Haykin, *Adaptive Filter Theory*, 5th ed. (Pearson Education, 2014).
5. V. I. Djigan, *Adaptive Filtering of Signals: Theory and Algorithms* [in Russian] (Tekhnosfera, Moscow, 2013).
6. O. L. Frost, "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE* **60**, No. 8, 926 (1972). DOI: 10.1109/PROC.1972.8817.
7. L. S. Resende, J. M. T. Romano, M. G. Bellanger, "A fast least-squares algorithm for linearly constrained adaptive filtering," *IEEE Trans. Signal Processing* **44**, No. 5, 1168 (1996). DOI: 10.1109/78.502329.
8. J. A. Apolinario, S. Werner, P. S. R. Diniz, T. I. Laakso, "Constrained normalized adaptive filters for CDMA mobile communications," *Proc. of 9th European Signal Processing Conf.*, 8-11 Sept. 1998, Rhodes, Greece (IEEE, 1998). URI: https://ieeexplore.ieee.org/document/7089625/.

9. M. R. L. De Campos, J. A. Apolinario, “The constrained affine projection algorithm - development and convergence issues,” *Proc. of First Balkan Conf. on Signal Processing, Communications, Circuits, and Systems*, Istanbul, May 2000 (2000).
10. A. Cantoni, P. Butler, “Properties of the eigenvectors of persymmetric matrices with applications to communication theory,” *IEEE Trans. Commun.* **24**, No. 8, 804 (1976). DOI: 10.1109/TCOM.1976.1093391.
11. R. Nitzberg, “Application of maximum likelihood estimation of persymmetric covariance matrices to adaptive processing,” *IEEE Trans. Aerospace Electronic Syst.* **AES-16**, No. 1, 124 (1980). DOI: 10.1109/TAES.1980.308887.
12. K.-C. Huarng, C.-C. Yen, “A unitary transformation method for angle-of-arrival estimation,” *IEEE Trans. Signal Processing* **39**, No. 4, 975 (1991). DOI: 10.1109/78.80927.
13. V. I. Zaritskii, V. N. Kokin, D. I. Lekhovitskii, V. V. Salamatin, “Recurrent adaptive processing algorithms under condition of central symmetry of space-time reception channels,” *Radiophys. Quantum Electron.* **28**, No. 7, 592 (1985). DOI: 10.1007/BF01034102.
14. K.-C. Huarng, C.-C. Teh, “Adaptive beamforming with conjugate symmetric weights,” *IEEE Trans. Antennas Propag.* **39**, No. 7, 926 (1991). DOI: 10.1109/8.86911.
15. L. Zhang, W. Liu, R. J. Langley, “A class of constrained adaptive beamforming algorithms based on uniform linear arrays,” *IEEE Trans. Signal Processing* **58**, No. 7, 3916 (2010). DOI: 10.1109/TSP.2010.2046078.
16. L. Zhang, W. Liu, R. J. Langley, “A class of constant modulus algorithms for uniform linear arrays with a conjugate symmetric constraint,” *Signal Processing* **90**, No. 9, 2760 (2010). DOI: 10.1016/j.sigpro.2010.04.003.
17. L. Zhang, W. Liu, R. J. Langley, “Adaptive beamforming with real-valued coefficients based on uniform linear arrays,” *IEEE Trans. Antennas Propag.* **59**, No. 3, 1047 (2011). DOI: 10.1109/TAP.2010.2103037.
18. L. Zhang, W. Liu, L. Yu, “Performance analysis for finite sample MVDR beamformer with forward backward processing,” *IEEE Trans. Signal Processing* **59**, No. 5, 2427 (2011). DOI: 10.1109/TSP.2011.2109957.
19. D. Xu, R. He, F. Shen, “Robust beamforming with magnitude response constraints and conjugate symmetric constraint,” *IEEE Commun. Lett.* **17**, No. 3, 561 (2013). DOI: 10.1109/LCOMM.2013.011513.122688.
20. M. V. Ratynskii and S. V. Petrov, “Realization of algorithms for stochastic signals in real-valued arithmetic,” *Tsifrovaya Obrabotka Signalov*, No. 4, 22 (2013). URI: https://elibrary.ru/item. asp?id=21197747.
21. J. Liu, W. Liu, H. Liu, B. Chen, X.-G. Xia, F. Dai, “Average SINR calculation of a persymmetric sample matrix inversion beamformer,” *IEEE Trans. Signal Processing* **64**, No. 8, 2135 (2016). DOI: 10.1109/TSP.2015.2512527.
22. D. I. Lekhovytskiy, “To the theory of adaptive signal processing in systems with centrally symmetric receive channels,” *EURASIP J. Adv. Signal Process.* **33**, 1 (2016). DOI: 10.1186/s13634-016-0329-z.
23. V. I. Djigan, “Odd symmetry of weights vector in linearly-constrained adaptive arrays with desired signal,” *Proc. of Int. Conf. on Antennas Theory and Techniques*, 24-27 May 2017, Kyiv, Ukraine (IEEE, 2017), pp. 140-144. DOI: 10.1109/ICATT.2017.7972604.
24. V. I. Djigan, “Algorithms for linearly constrained blind signal processing in digital antenna arrays with odd symmetry,” *Tsifrovaya Obrabotka Signalov*, No. 2, 3 (2015). URI: https://elibrary.ru/item.asp?id=23833950.
25. V. I. Djigan, “Multi-beam adaptive array,” *Izv. SFedU. Engineering Sciences*, No. 2, 23 (2012). URI: http://izv-tn.tti.sfedu.ru/?p=9412&lang=en.