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Method of Verification of Hypothesis about Mean Value on a Basis of Expansion in a Space with Generating Element

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Abstract—In this paper it is proposed an original method for verification of statistical hypotheses about mean values of random quantities. This method is based on Kunchenko stochastic polynomials tool and probabilistic description on a basis of higher order statistics (moments and/or cumulants). There are represented analytical expressions allowing to optimize decision rules using certain qualitive criterion and calculate decision-making error. It is shown polynomial decision rule in case of polynomial power $S = 1$ corresponds to classic linear decision rule which is used for comparative analysis. By means of multiple statistical experiments (Monte–Carlo method) obtained results of Neumann–Pierson criterion show proposed polynomial decision rules are characterized by increased accuracy (decrease of the 2nd genus errors probability) in compare to linear processing. The method efficiency increases with increase of stochastic polynomial order increase of degree of random quantities distribution difference from Gaussian probabilities distribution law.

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1. INTRODUCTION

It is known parametric approach to decision of the problem of statistic hypothesis verification allowing to obtain optimal solution is based on likelihood functional shaping which is calculated using empiric data. But practical implementation of such approach is accompanied by amount of information and computational problems. Moreover parametric methods can be characterized by essential complexity and inconvenience complicating their algorithmic realization and analysis of obtained solutions efficiency.

It leads to popularization of non-parametric statistics methods whose main advantage is their computational simplicity and absence of binding to specific distribution type. But “not accounting” of probabilistic properties of statistical data results in decrease of power of non-parametric criteria in compare to optimal parametric criteria.

One of compromise approach to statistic problems solution is building of probabilistic models on a basis of description of higher order statistics (moments or cumulants). Such description is incomplete, as a result there is only asymptotic possibility (with increase of parameters amount) of obtain of optimal methods of statistical processing on this basis. But its application is essentially more efficient from viewpoint of realization since it decreases essentially the requirements to a priori information and simplifies algorithmic realization of statistical processing methods.

The result is wide application of description tool of higher order statistics used for various problems solutions, for example, for detection and estimation of signals parameters [1–3], recognition and identification [4, 5], regression analysis [6–8], probabilistic and technical diagnosis [9–11], etc.

In this paper it is considered the problem of verification of simple hypothesis about mean value of random quantities. This problem represents one of the most typical examples of application of statistical hypothesis verification theory and it occurs in many fundamental papers of mathematical statistics [12, 13]. It can be explained by relative simplicity of such problems and great amount of application fields where this problem solution is required. In particular, such problems solution is key element for development of an algorithm for signals detection on a background of noise in statistical radio engineering [14], in communication field it is necessary for synthesis of procedures for recognition in detectors and demodulators of digital communication systems [15].

Examples of development of decision rules on a basis of functions of higher order sample statistic are experimental data approximation with Johnson’s [16] and Pierson’s [17] distributions for models with

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