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Mathematical Model of Digital Frequency-Sampling Filters with Shiftable Phase-Frequency Characteristic

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Abstract—The paper presents mathematical models of digital frequency-sampling filters with shiftable phase-frequency characteristic (PFC) and a generalized mathematical model taking into account the PFC shift introduced at the input. The conducted investigations made it possible to develop a class of narrow-band filters with the possibility of PFC shift and the real-time variation of instantaneous phase of the output signal.

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INTRODUCTION

Investigations of measuring self-oscillatory transducers of phase and frequency deviation operating in the enhanced sensitivity mode [1–3] revealed the need of synthesizing digital filters (DF) with tunable phase-frequency characteristic (PFC). It is well known that PFC of digital filters starts at the origin, while its slope is determined by the pulse response duration that complicates and in the majority of cases renders impossible the tuning of digital self-oscillatory system [4].

The performed investigations made it possible to synthesize a class of DF with shiftable PFC [5, 6] that allow us both to readjust PFC and have an effect on the instantaneous phase of signal subjected to processing [7]. This paper proposes a generalized mathematical model of DF with shiftable PFC.

MATHEMATICAL MODEL OF FREQUENCY-SAMPLING DF

The realization of any (arbitrary) algorithm of digital filtering is based on its mathematical description [8]. Let us consider different techniques of PFC shifting of digital FIR-filters based on the method of frequency sampling and sliding discrete complex Fourier transform. We shall obtain mathematical models for DF without PFC shift [9, 10] and DF with PFC shift.

Let us consider the operation of elementary digital filter (EDF) based on the sliding discrete complex Fourier transform [8, 9, 11, 12]. To this end, we shall derive an expression for finding the value of output signal sample using the known sliding sampling of input signal samples.

The output signal sample is formed on each quantization (sampling) interval by the signal last sample obtained as a result of the inverse Fourier transform [9]:

$$u_{\text{out}\,N-1} = \sum_{q=0}^{N-1} \underline{X}_q \cdot \mathrm{e}^{\frac{2\mathrm{j}\pi(N-1)q}{N}}, \tag{1}$$

where $\underline{X}$ are the complex spectrum samples of signal sliding sampling, u_{out} is the array of output signal samples, q is the number of spectral component of the Fourier series, N is the number of samples of the input signal sampling.

The frequency sampling method is numerically efficient as compared to the time convolution method at large values of N [8, 9]. In this case, factor $\exp(2\mathrm{j}\pi(N-1)q/N)$ can be assumed equal to unity. Numerical investigations show that the methodic simplification results in a minor PFC shift [8] and can be viewed as a

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