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Coherence Function of Interrelated Periodically Nonstationary Random Processes

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Abstract—A coherence function characterizing the correlation between harmonic components of two signals that are described by periodically correlated random processes has been proposed. Such function is shown to be invariant with regard to linear transformations of signals. A formula for coherence function is concretized for the amplitude- and phase-modulated signals.

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INTRODUCTION

The need of performing the cross-correlation and cross-spectral analysis of random processes arises during the analysis of single-channel and multi-channel data transmission systems, identification of signal propagation paths, localization of signal sources, etc. [1–3]. The quantitative description of stochastic interrelations of stationary random signals $\xi(t)$ and $\eta(t)$ implies the need of using a coherence function that is determined by the following expression [2, 3]:

$$\gamma_{\xi\eta}(\omega)=\frac{|f_{\xi\eta}(\omega)|}{[f_{\xi}(\omega)f_{\eta}(\omega)]^{1/2}}, \tag{1}$$

where $f_{\xi\eta}(\omega)$ is the cross-spectral density of stationary interrelated signals, $f_{\xi}(\omega)$ and $f_{\eta}(\omega)$ are their power spectral densities. Since relationship [2, 3]

$$|f_{\xi\eta}(\omega)|^2 \le f_{\xi}(\omega)f_{\eta}(\omega) \tag{2}$$

is valid, condition $0\le\gamma_{\xi\eta}(\omega)\le 1$ is always fulfilled for function (1). For independent signals equality $\gamma_{\xi\eta}(\omega)=0$ is valid for all $\omega\in R$. If signals $\xi(t)$ and $\eta(t)$ are the result of linear transformations of one and the same process, equality $\gamma_{\xi\eta}(\omega)=1$ is satisfied. If the coherence function is less than unity, the following situations are possible:

a) external noise affects one of the signals;

b) one of the signals underwent nonlinear transformations;

c) one of the signals is under the influence of other signals, except the signal under investigation.

In conducting the analysis of linear systems coherence function (1) makes it possible to separate that part of random signal $\eta(t)$ which is determined by process $\xi(t)$ at frequency ω. On the other hand, difference $1-\gamma_{\xi\eta}(\omega)$ characterizes that part of the spectrum which does not correlate with process $\xi(t)$.

The coherence function with similar properties can be expediently introduced to describe the stochastic interrelationship of two periodically correlated random processes (PCRP) representing a mathematical model that describes both stochasticity of signals and repeatability of their properties. The signals in communications, telemetry, radiolocation and hydrolocation systems acquire such properties in the process of modulation, scanning, coding, antenna rotation, etc. [4–6].