

# Interaction of High-Frequency and Low-Frequency Oscillations in the Synchronized Oscillator

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**Abstract**—Theoretical analysis of the oscillation synchronization under conditions of simultaneous low-frequency and high-frequency external impact on the oscillator has been conducted using an example of the generalized Van der Pol oscillator. It was shown that the interaction of high-frequency and low-frequency oscillations resulted in the appearance of additional regions of synchronization as a result of three-frequency interaction between the frequencies of such interaction and the natural frequency of oscillator. For the case of three-frequency interaction, characteristics of synchronous oscillations were determined and compared with those for the case of primary tone synchronization of oscillator.

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## INTRODUCTION

Synchronization of oscillations is referred to one of the main effects characterizing that characterizes the impact of external oscillation on the self-sustained oscillation system. The case of synchronization of self-sustained oscillators using an external harmonic input has been most fully studied in the classical synchronization theory [1–3]. In this case it is assumed that a self-sustained oscillator is synchronized if the following condition is satisfied:

$$\frac{\omega}{\omega_{\text{ext}}} = \frac{n}{m},$$

where  $\omega$  is the oscillation frequency,  $\omega_{\text{ext}}$  is the frequency of external input,  $n$  and  $m$  are integers. The case of  $n = m = 1$  corresponds to the primary tone synchronization.

In what follows, the notion of synchronization is extended to the case of interaction of chaotic behavior oscillators; the notion of partial synchronization is introduced, and in addition various factors affecting the synchronization of single and coupled self-sustained oscillation systems [4–6] are studied.

This paper considers with the problem of primary tone synchronization of self-sustained oscillator in the presence in the external action spectrum of an additional low-frequency component, the frequency of which does not satisfy the above specified resonance condition. A similar problem was considered earlier in [7–9] from the viewpoint of instabilities appearing in synchronized oscillators due to the development of chaotic oscillations.

Unlike the above specified papers, the current study considers the appearance of additional regions of existence of synchronous oscillations (emerging during the interaction of low-frequency and high-frequency oscillations) rather than the emergence of chaotic oscillations. As in the case of harmonic external signal, the synchronization of oscillator is considered to mean such mode of the excitation of oscillator where the spectrum of output oscillations contains the same spectral components and their linear combinations as the spectrum of external input.

The next section presents a mathematical model representing a generalized Van der Pol oscillator in the presence of external two-frequency input. This section also includes the derivation of corresponding abridged equations. The third section contains a brief review of the known results of analyzing the synchronous mode of oscillations in harmonically excited oscillator. The fourth section provides a description of three-frequency resonances in the analyzed oscillator that result in the appearance of additional regions of synchronization of oscillations. The characteristics of synchronous oscillations in