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Invariant of Group of Random Samples Mappings in the Sample Space with Lattice Properties

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Abstract—The characteristic of statistical interrelationship of random samples has been obtained, that is invariant with respect to the group of their mappings and that has been developed based upon the sample space with lattice properties. The possibility of its utilization for the analysis of quality of the images processing is being investigated.

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One of the significant problems of estimation of algorithms and devices for the image processing is the selection of an appropriate criterion for the evaluation of their effectiveness. In solving of some problems of image processing a part of the information, that is contained in them, is lost in some cases. This takes place in the problems of sampling and quantization of the analogue image signal, of image transformation using the techniques of spatial and/or frequency correction of images, of the reduction of color palette capacity (the dynamic range of the image), as well as of image compression. In order to estimate the quality of image processing the transformed from the source image is compared with the original one using any metric.

As a measure of proximity of the sample data x, y, which represent the pair of images X, Y, one utilizes the known metrics $d(x, y)$, that characterize the corresponding deviation measure for the values of the picture elements (pixels) $x_{ij} \in x, y_{ij} \in y$ [1–3]:

$$d_0(x, y) = \max_{i,j} |x_{ij} - y_{ij}|, \tag{1a}$$

$$d_p(x, y) = \left(\frac{1}{nm} \sum_i \sum_j |x_{ij} - y_{ij}|^p \right)^{1/p}, \quad p \in N, \tag{1b}$$

where N denotes the set of natural numbers; $x = \|x(i, j)\| = \|x_{ij}\|$, $y = \|y(i, j)\| = \|y_{ij}\|$ stand for the sample matrices of the two images X, Y; $i = 1, \ldots, n$, $j = 1, \ldots, m$; (i, j) designate the coordinates of the picture elements.

In practical applications one often utilizes the proximity measure, which is constructed based on the metric (1b) for $p = 2$, that is designated as peak signal-to-noise ratio (PSNR) [3]:

$$d_{\text{PSNR}}(x, y) = 10 \lg \left(\frac{A^2 nm}{\sum_i \sum_j |x_{ij} - y_{ij}|^2} \right), \tag{2}$$

where A stands for the white level.

In some problems of signal processing one uses such proximity measures between the random variables ξ, η as correlation coefficient $\rho_{\xi\eta}$:

$$\rho_{\xi\eta} = \rho(\xi, \eta) = \frac{\mathrm{M}[(\xi - m_\xi)(\eta - m_\eta)]}{\sqrt{\mathrm{D}(\xi)\mathrm{D}(\eta)}}, \tag{3}$$