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Class of Non-Gaussian Distributions with Zero Skewness and Kurtosis

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Abstract—A mathematical model of non-Gaussian distributions with zero skewness and kurtosis has been defined. This model represents a class of two-component mixtures of conjugate distributions with equal weight coefficients. An equation was obtained that the second and fourth initial moments of the mixture components should satisfy. Examples of non-Gaussian distributions with zero skewness and kurtosis were considered. These examples showed that the sixth and eighth cumulant coefficients in such distributions could be positive, negative or nonexistent. The obtained results make it possible to perform the mathematical and computer simulation of non-Gaussian distributions with zero skewness and kurtosis.

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INTRODUCTION

The solution of many problems based on the probabilistic approach involves the need of knowledge of the distribution function of random quantity or random process under investigation. Such applied problems include the determination of measurement accuracy [1], finding of detection characteristics [2], the decision rules and classification errors [3], etc.

In the majority of practical cases the derivation of exact analytical expressions of distribution functions of random quantities or random processes under investigation is not feasible [4, 5], therefore, various approximated distribution functions are used. At present the main approximating distribution is Gaussian, the application of which is usually substantiated by the central limit theorem. Among the other approximating distributions the systems of Pearson and Johnson distributions and the sections of orthogonal series are most often used [1, 4–9].

The choice of a particular approximating distribution is based on the method of moments and usually makes use of cumulant coefficients γ_s:

$$\gamma_s = \frac{\kappa_s}{\kappa_s^{s/2}}, \tag{1}$$

where κ_s are the distribution cumulants determined by formula [9]:

$$\kappa_s = \left.\frac{\mathrm{d}^s \ln f(u)}{\mathrm{i}^s \mathrm{d}u^s}\right|_{u=0},$$

$f(u)$ is the characteristic function, $\mathrm{i} = \sqrt{-1}$.

It should be emphasized that for the Gaussian distribution the following relationships are valid:

$$\gamma_s = 0, \quad s \geq 3. \tag{2}$$

The choice of approximating distributions from the Pearson and Johnson systems is performed on the basis of values of skewness γ_3 and kurtosis γ_4 coefficients without regard to the rest of cumulant coefficients. In particular, in these systems condition