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Least Squares Method in the Statistic Analysis of Periodically Correlated Random Processes

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Abstract—The properties of least-squares estimates of mathematical expectations and correlation function of periodically correlated random processes (mathematical model of stochastic oscillations) have been investigated. The formulas defining the statistical characteristics of estimates were analyzed. In addition, examples were presented for illustrating the analysis of modulated signals.

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Periodically correlated random processes (PCRP) represent a mathematical model for a wide range of physical phenomena, since they describe both the repeatability and the stochasticity of temporal variability [1–5]. Taking account of the periodic correlation of signals applied in telecommunications and telemetry makes it possible to solve more effectively the problems of signal analysis, transformation, and processing [3, 4]. The analysis of vibration signals based on a model in the form of PCRP enables us to improve the efficiency of diagnostics, in particular uncover defects of mechanisms at early stages of their development [6, 7]. Mathematical expectation of PCRP $m(t) = E\xi(t)$, distribution averaging operator E, and also correlation function $b(t,u) = E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+u)$, $\overset{\circ}{\xi}(t) = \xi(t) - m(t)$ are periodic functions of time t and therefore can be presented by the Fourier series:

$$m(t) = \sum_{k\in\mathbb{Z}} m_k e^{ik\omega_0 t}, \quad b(t,u) = \sum_{k\in\mathbb{Z}} B_k(u) e^{ik\omega_0 t},$$

where $\omega_0 = 2\pi/T$, T is the period. The objective of the correlation statistical analysis is to determine functions $m(t)$ and $b(t,u)$ (as functions of two variables: time t and shift u) and also their Fourier coefficients m_k and $B_k(u)$ (the latter are also called correlation components) on the basis of experimental data. Such determination can be performed by using either coherent [8] or component [9] methods. The first of them is based on averaging the readings of process realization over period T:

$$\hat{m}(t) = \frac{1}{N}\sum_{n=0}^{N-1} m(t+nT),$$

$$\hat{b}(t,u) = \frac{1}{N}\sum_{n=0}^{N-1}\left[\xi(t+nT) - \hat{m}(t+nT)\right]\left[\xi(t+u+nT) - \hat{m}(t+u+nT)\right],$$

here N is the number of periods that are averaged, while the second method is based on using the trigonometric interpolation

$$\hat{m}(t) = \frac{1}{N}\sum_{k=-N_1}^{N_1}\hat{m}_k e^{ik\omega_0 t},$$