Statistical Characteristics of Soft Decoding Transformation

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Abstract—On a basis of numerical results of imitation simulation there are considered basic statistical characteristics of on-linear transformation of soft decoding for practically applied approximation of logarithmic likelihood ratio of binary sums of code symbols from random values of continuous (quantized) measurements.

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PROBLEM STATEMENT

One of the main direction of perfection of error correction coding system is application of methods, realizing a conception of “soft input–soft output”. In binary systems with soft output there are realized transformations of sets of continuous (quantized) instantaneous values \( z_i \) of receiver signals into continuous values \( x_i \) of a posterior statistics of information symbols, which are non-linearly related to random measurements [1, 2].

With purpose of calculation burden decrease there are used different approximations of non-linear relations of a posteriori statistics of information symbols \( x_i \) and measurements \( z_i \). At that, as a rule, it is considered only approximation accuracy (errors), and basic statistical characteristics of non-linear transformation results for random measurements realized approximations are not analyzed. But statistical characteristics of random measurements non-linear transformation \( z_i \) for different approximation define efficiency of exchange algorithms of soft decoding.

Therefore we consider some statistical characteristics of non-linear transformation of soft decoding for basic used approximations of dependences of logarithmic likelihood ratio values for binary sums of code symbols.

MAIN PART

The basis of all methods of soft decoding [1, 2] is non-linear transformation relation of values \( x_1, x_2 \) of log likelihood ratio (LLR) of independent equiprobable code symbols with their binary sum LLR value \( L(x_1 \oplus x_2) \)

\[
L(x_1 \oplus x_2) = \ln \frac{1 + \exp(x_1 + x_2)}{\exp(x_1) + \exp(x_2)}
\]

(1)

Transformation relation (1) is a basic one for symbolwise decoding of linear codes according to maximum of a posteriori probability (MAP), therefore non-linear function in right part of relation is called LLR function of MAP algorithm. Essentially non-linear behavior of this transformation is illustrated in Fig. 1a; the shape of function of MAP algorithm is “butterfly”.

Different approximation of LLR function of MAP algorithm (1) can be used for practical tasks of soft decoding. Essential decrease of computational burden is achieved in Log-MAP decoding algorithms in case of application of logarithm Jacobian determinant

\[
\ln(1 + \exp(x_2 + x_1)) \equiv \max(0, x_2 + x_1) + \ln(1 + \exp(-|x_2 + x_1|)),
\]

\[
\ln(\exp(x_1) + \exp(x_2)) \equiv \max(x_1, x_2) + \ln(1 + \exp(-|x_2 - x_1|)).
\]