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Sigma-Delta Modulator: Refinement of Equivalent Circuit and Transfer Function

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Abstract—Analysis of sigma-delta modulator and its models (equivalent circuits) is performed. New refined equivalent circuit of modulator and its transfer function are suggested. Modeling of modulator is performed which confirms results of analysis.

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Sigma-delta modulator is widely used in transformation and signal transmission systems [1–4]. Two concepts of representing this modulator are well-known in the form of models (equivalent circuits) used for its analysis and design [1, 2]. To the authors' mind, both concepts contain errors.

The work aims to provide analysis of equivalent circuits and to suggest a new refined circuit and, correspondingly, refined transfer function of modulator. Besides, authors will perform "independent" modeling of modulator to confirm results of analysis.

Sigma-delta modulator belongs to pulse modulators together with pulse-frequency (FPM) [5] and pulse-duration (PDM) modulators. Modulators may generate signals with both continuous (continuous PFM, PDM) and with quantized (quantized PFM, PDM) change of parameter. Signal of sigma-delta modulation (SDM signal) is a quantized version of PFM signal, but differs from the latter in one more positive property, considered below. All mentioned signals, besides modulation of the corresponding parameter, can be characterized by presence of additive analogue modulating signal which may be restored using filtration.

A structure diagram of SDM which contains input summing (differential, to be more precise) circuit (Σ), integrator ($\int$), comparator ($\ulcorner$), output pulse duration shaper (PDS), controlled by clock pulses (f_{cl}) and pulse area shaper (PAS) in the negative feedback circuit is depicted in Fig. 1.

PDS forms pulses and intervals between them, whose duration is equal or multiple to quantization period

$$\tau = 1 / f_{cl} = \text{const}, \tag{1}$$

where f_{cl} is the frequency of clock pulses (quantization frequency), while PAS additionally normalizes pulses of modulator with respect to amplitude, generating "quanta" of area of the same duration. "Quantum's" area is equal

$$S_0 = 2E_0\tau = \text{const}, \tag{2}$$

where E_0 is bias voltage (Fig. 1), equal in this case to the half of the band of modulator's input voltages, and, correspondingly, to the half of "quantum's" amplitude. PDS and PAS may be a single element, whose output is modulator's output. Besides together with comparator they may represent a single functional element—quantizer shown below as part of the refined equivalent modulator's diagram. PFM possesses a similar structure diagram as in Fig. 1 but without synchronization circuit by clock pulses (f_{cl}).

Graphs of voltages in circuits of PFM (a), (b) and SDM (c)–(g) are shown in Fig. 2. Pulses of PFM signal formed without synchronization by clock pulses are shown as well (for visualization) with duration equal τ. Voltages of integrators (with thresholds of comparators shown by thin horizontal lines) and output pulse sequences are depicted in Fig. 2a, d, f and Fig. 2b, e, g, respectively. Clock pulses used for synchronization of SDM are symbolically depicted with short pulse time (Fig. 2c).