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Continued Fractions in Time Varying Circuits Analysis

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Abstract—Analysis of linear radio circuits with time varying parameters results in infinite system of algebraic equations. Theory of such equations systems with necessary for practice fullness has not been developed. Therefore special case of time varying circuit is of great interest when its analysis is simpler and is possible to be reduced to continued fractions, in spite of all this fundamental characteristics of time varying circuit are preserved.

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Now nonlinear systems in whole, and radio electronics systems in particular, are of great interest. Current theory of nonlinear circuits [1] is insufficiently methodical, and it does not satisfy practical necessities. Linear connection principle [2] states that solution of nonlinear equation can be realized in specially selected linear equation. It appears arbitrary process in nonlinear radio circuit can represented by special linear circuit with the same process. Hence, there is a new theory of general circuits (time varying circuits). It is methodical introduction in nonlinear circuits theory. Adequate mathematic tools of time varying circuits are enough complicated [3, 4]. In this connection mathematically “the simplest” case, which, however does not decrease of generality of processes in time varying circuits, is of great interest. Analysis of time varying circuits is reduced to infinite system of algebraic equations [5–8]. Natural method of these equations is a reduction method, but it can be applied only in case of convergence of equations system [9], but in general case it is not proven. It is advisable to consider analysis problem with regard to simple model of time varying circuit, at that it is necessary to select “the simplest” in some meaning case. Now we show that in this case a problem is reduced to continued fractions [10, 11]. Let we consider sequential oscillating circuit (Fig. 1).

For our purposes it is convenient to introduce instead of capacity C a stiffness $S = 1/C$. Initial assumptions are following: all circuit elements are positive and time varying functions:

$$\begin{aligned} L(t) &= L_0[1 + m_L \cos(\Omega t + \varphi_L)], \\ S(t) &= S_0[1 + m_S \cos(\Omega t + \varphi_S)], \\ R(t) &= R_0[1 + m_R \cos(\Omega t + \varphi_R)]. \end{aligned} \tag{1}$$

The circuit is excited by harmonic emf

$$\varepsilon(t) = E\cos(\omega t + \varphi). \tag{2}$$

Coefficients of parameters modulations are supposed lesser than one

$$m_L,\ m_S,\ m_R < 1.$$

Equation of forced oscillations is obtained from the second Kirchhoff’s law

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + (R + \dot{L})i + S\int i\mathrm{d}t = E\cos(\omega t + \varphi). \tag{3}$$

Here point at the top means time differentiating.