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Analysis of Free Processes in Time Varying Circuit by Means of Generalized Characteristic Equations Method

V. V. Beloglazov, N. D. Biryuk, and V. V. Yurgelas

Voronezh State University, Voronezh, Russia

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Abstract—Developing methods of analysis of free processes in time varying circuit is important both for time varying and nonlinear radio circuits. The principle of linear connection brings together processes in time varying and nonlinear circuits. It is of special interest to extend the existing methods developed for radio circuits with stable parameters to time varying circuits. This article is devoted to one of such methods—the method of characteristic equations for differential equations with fixed factors.

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A problem of analysis of free oscillations in time varying circuit appeared long ago [1], firstly, it was reduced to Mathieu equation as the most developed second order linear differential equitation with parametrical coefficient. Further there was developed principle of linear connection [2, page 159], which states arbitrary process in non-linear system can be realized in specially searched linear system. Principle of linear connection implies arbitrary contribution in parametrical system theory is essential also for non-linear systems. Methods of general form analysis are the most interesting, since Mathieu equations theory is insufficient due to is partial character. Characteristic of general methods of analysis of time varying circuit is represented in papers [3, 4], and also in monographs [5–8]. Very promising is generalizing of known methods, developed for analysis of differential equations with constant coefficients to linear differential equations of general form. This scientific direction was developed by collaborators of Moscow Aviation Institute headed by F. A. Mikhaylov. Main achievements are generalized in monograph [9], where complete bibliography of this direction papers is contained. But radio electronic problems have their specific features, and mentioned direction were developed for application in automated control problems. It is advisable to spread corresponding achievements to radioelectronic oscillating systems. Now we represent such generalization to structurally simple but essentially complex oscillating system—time varying circuit.

The most spread is simple circuit of time varying series circuit (Fig. 1).

It is supposed that circuit elements: capacity $C(t)$, resistance $R(t)$ and inductance $L(t)$ are time varying according to arbitrary functions with the single restriction, which is existence of time derivative $\dot{L} = \mathrm{d}L / \mathrm{d}t$. We suppose there are defined some initial conditions, i.e. at time point $t = 0$ capacitor charge is q_0 and (or) inductance current is i_0. It is necessary to obtain a function $q = q(t)$ for free charge oscillations. This problem in general statement is complex and can not be expressed with elementary functions. Theoretically exact solution can be obtained by means of application of infinite calculations. A problem is to obtain approximate solution with required accuracy by means of finite amount of calculation (which is desired to be small).

The second Kirchhoff law leads to free process equation, which is advisable to be represented according to capacitor charge q:

$$L(t)\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + [R(t) + \dot{L}(t)]\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{C(t)}q = 0. \tag{1}$$

This is an equation of following type

$$a_2(t)\ddot{x} + a_1(t)\dot{x} + a_0(t)x = 0. \tag{2}$$

As it follows from general theory [10] of this equation, if at least one its nontrivial solution $x_0 = x_0(t)$ is known then generalized solution of this equation can be built