ISSN 0735-2727, Radioelectronics and Communications Systems, 2009, Vol. 52, No. 10, pp. 561–564. © Allerton Press, Inc., 2009.
Original Russian Text © V.I. An, 2009, published in Izv. Vyssh. Uchebn. Zaved., Radioelektron., 2009, Vol. 52, No. 10, pp. 67–71.

Phase Derivative Distribution of Signal and Gaussian Noise Sum

V. I. An
Moscow State Technical University n.a. N.E. Bauman (MSTU), Moscow, Russia
Received in final form October 27, 2008

Abstract—Characteristic function and integral function of the phase derivative distribution of harmonic signal and narrowband Gaussian noise sum are obtained for the case when the signal's central frequency coincides with the central frequency of noise spectrum. It is shown that the power series for the distribution density of the phase derivative is determined by odd moments of the envelope.

DOI: 10.3103/S0735272709100082

For the sum of harmonic signal and narrowband Gaussian noise, when the central frequency of signal coincides with the central frequency of symmetrical noise spectrum, the distribution density of the phase derivative $\psi' = \psi'(t)$ at the moment in time t is known [1]:

$$p(x) = \frac{1}{2(1+x^2)^{3/2}} \exp\left(-\frac{a^2}{2}\right) {}_1F_1\left[\frac{3}{2}, 1, \frac{a^2}{2(1+x^2)}\right], \tag{1}$$

where $x = \dfrac{\psi'}{\sigma_1/\sigma}$, $a = \dfrac{A_0}{\sigma}$, $\sigma_1 = \sigma\omega_*$, σ^2 stands for noise dispersion, ω_* means root-mean-square noise spectrum bandwidth, A_0 is the signals' amplitude.

In the scientific-technical literature the other characteristics of the phase derivative distribution are absent, which limits the methods of conducting research and solving problems, which deal with its application. Below some of these characteristics are obtained using distribution density transformation (1).

In [2] it is shown, that the phase derivative in the case when signal's frequency coincides with the central frequency of noise appears to be a quotient of dividing independent random variables $Y \in (-\infty, \infty)$ and $Z \in (0, \infty)$ by the distribution densities respectively

$$p_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right), \tag{2}$$

$$p_Z(z) = z \exp\left(-\frac{z^2 + a^2}{2}\right) I_0(za). \tag{3}$$

Thus, (1) may be rewritten in the following form:

$$p(x) = \int_0^\infty p_Y(xz) p_Z(z) z \mathrm{d}z = \int_0^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 z^2}{2}\right) z^2 \exp\left(-\frac{z^2 + a^2}{2}\right) I_0(za) \mathrm{d}z. \tag{4}$$

Let's determine the characteristic function of the phase derivative $\Theta(u)$ from (4). To accomplish that we shall use the Fourier transformation of normal distribution [1]