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Distribution of Time Intervals Determining the First Instant when a Stationary Process Reaches the Set Level

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Abstract—This paper describes a method of determining the rigorous analytical expression for the distribution of time instants when a stationary process differentiable in the root-mean-square sense attains threshold for the first time. It has been shown that this expression is a solution of the partial integral-differential equation corroborating earlier results.

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The first attempt of finding the distribution of intervals between zeros of the stationary random process was undertaken by S. Rice in paper [1]. The investigation method was based on the technique of "inclusion and exclusion" of zeros in the interval between certain initial time of analysis t_0 and time $t_0 + t$. The basic principle represents the rule for probabilities of the sum of events implying that

$$P\{A \cup B\} = P\{A\} + P\{B\} - P\{A \cap B\},$$

$$P\{A \cup B \cup C\} = P\{A\} + P\{B\} + P\{C\} - P\{A \cap B\} - P\{A \cap C\} - P\{B \cap C\} + P\{A \cap B \cap C\},$$

etc. The author of the paper without any superfluous details gives the final result. If $Q_\theta(h,t)$ designates the distribution of intervals θ (Fig. 1) between the surges of stationary process above the level, the probability of the process having a zero with negative slope at point t_0, zero with positive slope at point $t_0 + \theta$, while zeros are not present in the interval $[t_0, t_0 + \theta]$ will be equal [1, p. 144]:

$$p_0(t)[1 - Q_\theta(0,t)] = p_0(t) - \frac{1}{1!}\int_{t_0}^{t_0+t} p_1(t,t_1)\mathrm{d}t_1 + \frac{1}{2!}\int_{t_0}^{t_0+t}\int_{t_0}^{t_0+t} p_2(t,t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2$$

$$-\frac{1}{3!}\int_{t_0}^{t_0+t}\int_{t_0}^{t_0+t}\int_{t_0}^{t_0+t} p_3(t,t_1,t_2,t_3)\mathrm{d}t_1\mathrm{d}t_2\mathrm{d}t_3 + \ldots, \tag{1}$$

where $p_0(t)$ is the probability of the event that the process has a zero with negative slope on interval dt; $p_1(t,t_1)$ is the probability of the event that the process has a zero with negative slope on interval dt and with positive slope on interval dt_1; $p_2(t,t_1,t_2)$ is the probability of the event that the process has a zero with negative slope on interval dt and with positive slope on intervals dt_1 and dt_2, etc.

Expression (1) represents an infinite slowly convergent series of integrals of increasing multiplicity that only in the case of independent determination of zeros on intervals dt, dt_1,…, dt_n leads to a simple result $1 - Q_\theta(0,t) = \mathrm{e}^{-\nu t}$, where ν is the probability of the event that the process has a zero with positive slope on intervals dt_i, $i = 1\ldots n$, $n = \infty$. In other cases the computation of only first several terms of series (1) runs into insurmountable difficulties even with the use of high-speed computers.

A more detailed derivation of the distribution of time intervals determining the first instant of attaining a certain threshold is presented in papers of B.R. Levin [2] and Ya.A. Fomin [3], and also in their joint papers [4, 5].