

Family of Two-Dimensional Correcting Codes on a Basis of Perfect Binary Array

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Abstract—We consider a family of two-dimensional correcting $E(N)$ -codes on a basis of perfect binary arrays $H(N)$, and we show correcting possibilities of $E(N)$ -codes in comparison with corresponding BCH-codes with maximal length are essentially better with regard to correction of package (correlated) errors, at that, uncorrelated errors are corrected identically.

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Perfect binary arrays (PBA) are described in many papers and they are used in many fields of application [1–7]. Main purpose of the paper is research of correction possibilities of two-dimension cyclic codes on a basis of PBA.

Let $H_0(N) = \|h_{i,j}\|$, $i = \overline{1, N}$, $j = \overline{1, N}$, $h_{i,j} \in \{-1, +1\}$ is N th order PBA, where $N = 2^k$ or $N = 3 \times 2^k$. We represent two-dimension periodic auto-correlation function (TPACF) of this array in matrix form:

$$R(m, n) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} h_{i,j} h_{i+m, j+n} = \text{tr}[H_0 \cdot (L_m H_0 Q_n)^T] = \begin{cases} N^2, & m = n = 0, \\ 0, & \text{other } m \text{ and } n, \end{cases} \quad (1)$$

where L_m and Q_n are array cyclic shifts operators, correspondingly, by rows (down) and columns (to the left) on a value $m = 0, N-1$, $n = 0, N-1$, $\text{tr}(A)$ is matrix A trace.

For example, one of arrays, which is a part of total class of PBA [6], represented in sign form $\{\pm 1\} \Rightarrow \{\pm\}$, and its TPACF are following:

$$H_0(6) = \begin{bmatrix} + & + & + & + & + & - \\ + & - & + & + & - & + \\ - & + & - & + & - & - \\ + & + & - & - & + & + \\ - & + & - & + & - & - \\ - & + & + & + & + & - \end{bmatrix}, \quad \|R(m, n)\| = \begin{bmatrix} 36 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2)$$

Two-dimension $E(N)$ -code is built on a basis of randomly selected (reference) array $H_0(N)$ by means of all cyclic shifts on rows and (or) columns. As it follows from the definition, volume of $E(N)$ -code is $J = N^2$. We consider structural and correlation properties of two-dimensional and correspondent one-dimension code words of $E(N)$ -code.

We unroll sequentially by rows each array of $E(N)$ -code, then we obtain one-dimension orthogonal non-cyclic $h(J)$ -code of J order, which has a property of two-loop cyclic N -shift [8]. Hence, $h(J)$ -codes can be efficiently decoded by means of fast Vilenkin–Krestenson transformation, calculating N -convolutions or dyadic convolutions (correlations) [8].

In this paper main attention is paid to decoding of two-dimension $E(N)$ -codes on a basis of accounting of two-dimension periodic cross-correlation functions (TPCCF) arrays of $E(N)$ -codes. We note that TPCCF of