ISSN 0735-2727, Radioelectronics and Communications Systems, 2009, Vol. 52, No. 5, pp. 248–255. © Allerton Press, Inc., 2009.
Original Russian Text © A.A. Popov, 2009, published in Izv. Vyssh. Uchebn. Zaved., Radioelektron., 2009, Vol. 52, No. 5, pp. 34–45.

Characteristics and Properties of Signals Space Built on Generalized Boolean Algebra with Measure

A. A. Popov
National Academy of Defence, Kyiv, Ukraine
Received in final form May 5, 2008

Abstract—The characteristics of an arbitrary pair of random processes, which are invariant with respect to the group of random process transformations, are considered. By applying an apparatus of Boolean algebra with measure to the introduced characteristics of random processes, a notion of metric signals space with axiomatic introduction of information quantity measure, which allows to estimate information ratios between random signals and their instantaneous values, is formulated.

DOI: 10.3103/S0735272709050045

Analysis of development of signals theory and information theory unintentionally leads to a thought of their rather independent and isolated existence with quite weak penetration into each other. Frequently it may seem that information carriers—signals and principles of their processing exist on their own, while the information carried by their means and approaches to describing a wide range of information processes are isolated from the information carriers. In part this is caused by the fact that the authors of signals theory and information theory considered, on one hand, a space of signals irrespectively of information carried by the signals in their works [1, 2], but, on the other hand, a measure of information quantity was considered irrespective of its material carriers—signals [3–5]. This controversy inevitably earlier or later poses a question of necessity to unify mathematical basics of signals processing theory and information theory. In this work the author shows that this theoretical difficulty may be dealt with by means of applying the apparatus of Boolean algebra with measure to describing the probability-statistical characteristics and properties of random signals.

To describe probability characteristics of a pair of random processes $\xi(t)$ and $\eta(t')$ we shall use various denotions of the time parameter t and t', assuming that these random processes, in a general case, exist in separate reference systems.

In order to determine, on one hand, a full measure of statistical interconnection between two instantaneous values (samples) of $\xi(t_j)$ and $\xi(t_k)$ of the random process $\xi(t)$, and, on the other hand, an invariant of mutually univocal mapping of a random process, a normalized function of statistical interconnection (NFSI) is introduced in paper [6]:

$$\psi_\xi(t_j,t_k) = [1-(1-d(p,p_0))^2]^{1/2},$$

where $d(p,p_0)$ stands for a metric between two-dimensional probability distribution density (PDD) $p_\xi(x_j,x_k;t_j,t_k)$ of the pair of samples $\xi(t_j)$ and $\xi(t_k)$ and product of their single-dimensional PDDs $p_\xi(x_j;t_j)$ and $p_\xi(x_k;t_k)$:

$$d(p,p_0) = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|p_\xi(x_j,x_k;t_j,t_k) - p_\xi(x_j;t_j)p_\xi(x_k;t_k)\right|\mathrm{d}x_j\mathrm{d}x_k.$$

Index “0” in p_0 reflects the peculiarity that the product of single-dimensional PDDs corresponds to the two-dimensional PDD of statistically independent pair of samples. The main properties of NFSI are considered in work [6].

Some function $i_\xi(t_j,t)$ may be put into correspondence to each time sample $\xi(t_j)$ of random process $\xi(t)$ with NFSI $\psi_\xi(t_j,t_k)$. We shall call this function information distribution density (IDD), which is connected with NFSI through the following relation