
Processing Characteristics of Harmonic Signals against Noises in Case of Their Interaction in K-Space

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Abstract—In this paper it is shown that processing of harmonic signals with unknown nonrandom amplitude and phase, against noises in K -space allows achieving better processing characteristics than ones in linear space. It is proposed signals processing quality criteria, based on metrical properties of an estimation space.

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The most common problem of processing of signals against noises is a problem of signals and its parameters estimation. This problem is closely related to other similar ones: signal recognition, detection and determination [1–4]. Many authors propose to process signals against noises in linear space, using signals $L(+)$. In this space resulted signal x is an additive mixture of an abelianized group of signal s and noise n : $x = s + n$.

We propose to process harmonic signals against noises in K -space. K -space properties and characteristics are well known and described in many algebraic papers, such as [5–7]. K -space application as a signal space allows using both algebraic lattice axiomatic and linear space axiomatic. K -space is defined as a linear distributive lattice $K(+, \vee, \wedge)$ above scalars ring, where axioms of algebraic distributive lattice $K(\vee, \wedge)$ with upper and lower boundaries operations: $a \vee b = \sup_K a, b$, $a \wedge b = \inf_K a, b$ and axioms of linear space L with operations of additive addition $a + b$ for abelianized group $L(+)$ can be used [5, 6]. Algebraic lattice absorption $s \wedge (s \vee n) = s$, $s \vee (s \wedge n) = s$ provides recognition of known signal in case of its interaction with noise in K -space without any loss caused by noises. However, this approach cannot be used in case of unknown signals. We research interaction in K -space of harmonic signal with unknown nonrandom amplitude and initial phase and Gaussian noise $x(t) = s(t) \oplus n(t)$, where operation \oplus corresponds to one of two binary operations \vee, \wedge of the algebraic lattice K : $s \vee n = \sup_K \{s, n\}$, $s \wedge n = \inf_K \{s, n\}$.

Received signal $s(t)$ model is:

$$s(t) = A \cos(\omega_0 t + \varphi), \quad t \in T_s = [t_0, t_0 + T],$$

where A is unknown nonrandom amplitude of useful signal s ; φ is unknown nonrandom initial phase of useful signal, $\varphi \in [0, 2\pi]$; $t_0 = 0$ is known signal arrival time; T is a duration of the signal.

Narrow-band Gaussian noise $n(t)$ model is:

$$n(t) = A_n(t) \cos(\omega_0 t + \varphi_n(t)),$$

where где $A_n(t)$ is noise stochastic envelope with Rayleigh distribution, $\varphi_n(t)$ is stochastic noise phase with uniform distribution at the interval of $[0, 2\pi]$.

Processing device must fulfill Hilbert transformation of signals sum $s + n$ and create two processing channels for measuring of unknown nonrandom harmonic signal amplitude and initial phase. Let we have a device $C [+/\vee\wedge]$ which can provide inter-unilateral mapping of the linear space $L(+)$ with additive sum operation “+” (sum of the signal s and the noise n) into K -space taking into account identity [5–7]: $s + n = (s \vee n) + (s \wedge n)$. Convector $C[+/\vee\wedge]$ provide mapping of linear space signals $L(+)$ into K -space signals in each $\cos(\sin)$ -quadrature channel: