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Stability Criteria for Parametric System Consisting of Two Coupled Circuits with External Conductor Link

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Abstract—A more general Lyapunov function has been proposed in the form of instantaneous energy stored in reactances that allowed us to obtain several new stability criteria for a parametric system consisting of two coupled circuits with external conductor coupling.

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Free process of the electric oscillating system the diagram of which is presented in Fig. 1 was considered earlier in paper [1]. All elements of the system (capacitances, inductances, and resistances) are assumed to be positive and time-varying in an arbitrary way, but irrespective of the currents passing through them. The only restriction is that reactances must have the first time derivative. As can be seen from the diagram, the system consists of two oscillating circuits coupled between themselves. The coupling element is active resistance R.

From physical considerations it follows that thermal losses show with different intensity over the entire system. Usually, for the sake of simplicity they are taken into account by using lumped active resistances and conductances. The smaller are the thermal losses, the higher is the accuracy of this approximation. As a rule, thermal losses in parametric systems are higher than in the appropriate systems with constant parameters. In the diagram under consideration the thermal losses are accounted for in the form of appropriate resistors. As follows from their physical nature these thermal losses are taken into account separately for capacitances and inductances, and also they can be artificially introduced into any part of the radio circuit by connection of active resistances. Resistances $R^{(1)}$ and $R^{(2)}$ take into account internal losses of power sources that may be connected to appropriate points of our system.

If the initial conditions are specified in reactances, a free process shall exist in the system. In the system with constant elements in accordance with Fig. 1, the free process is known to be exponentially decaying. It tends towards zero with infinitely increasing time. In the case of the parametric system shown in Fig. 1 the problem of free process gets radically complicated. It may tend to zero as in the previous case, then the system is Lyapunov asymptotically stable. It may occur that the free process tends neither to zero nor to infinity, but oscillates between two finite values, then the system is Lyapunov stable. Finally, it is possible that the free process with the infinite rise of time tends to infinity, then the system is Lyapunov unstable. It turns out that the problem of differentiation of the above cases presents some difficulties and cannot be accurately solved yet. However, using the Lyapunov second method it is possible to obtain the sufficient conditions of stability of the oscillating system. This is related to building Lyapunov's special function. Since the Lyapunov function can be specified ambiguously, it appears that for each Lyapunov function special sufficient stability conditions has to be proved, the relationship between which is difficult to determine. This problem is considered below in detail.

Mathematical description of the free process of the oscillating system in Fig. 1 is obtained on the basis of Kirchhoff's first and second laws. In this case the determining functions of the process should be selected. The charges of capacitors and magnetic fluxes coupled with inductances are traditionally used as the above functions. In our case the same option is chosen: charges are designated as q_1 and q_2, while the magnetic fluxes—as Φ_1 and Φ_2. The free process is represented by a linear system of four first-order differential equations. The system is linear because its elements are assumed to be time variable irrespective of the currents passing through them.

In this case this system of equations assumes the form