ISSN 0735-2727, Radioelectronics and Communications Systems, 2008, Vol. 51, No. 7, pp. 368–376. © Allerton Press, Inc., 2008.
Original Russian Text © A.A. Popov, 2008, published in Izv. Vyssh. Uchebn. Zaved., Radioelektron., 2008, Vol. 51, No. 7, pp. 29–40.

A Comparative Analysis of Estimates of the Unknown Nonrandom Parameter of Signal in the Linear Space and K-Space

A. A. Popov
National Academy of Defence, Kyiv, Ukraine
Received in final form February 27, 2007

Abstract—It has been shown that signal processing against the background of interferences (noises) in the *K*-space of signals makes it possible to achieve higher characteristics of the estimation of unknown nonrandom parameter of signal than those in the linear space. Quality index of the point estimation based on metric properties of the estimation space has been proposed.

DOI: 10.3103/S0735272708070030

One of the most general problems of signal processing against the background of interferences (noises) is the estimation of signals and their parameters. Other problems, e.g., the problems of signal detection, differentiation and resolution of signals can be also reduced to this problem [1–4]. In a greater part of available literature the problems of signal processing against the background of interferences (noises) are stated in terminology of the linear space of signals £, where the result of interaction x of signal s and interference n is described by the addition operation of additive commutative group: $x = s + n$.

Characteristics and behaviour of estimates under an assumption of additive (in the terminology context of the linear space) interaction of the estimated parameter with measurement errors of a certain arbitrary family of distributions are sufficiently amply presented in the relevant literature [5–9]. Examples of the estimates, the asymptotic dispersion of which never exceeds the Cramer-Rao lower bound, while at certain values of the estimated parameter turns out to be lower than this bound, were offered by J. Hodges and L. Le Cam [5, 8]. Such estimates are commonly called superefficient. This paper presents an example of estimates that are close to superefficient in terms of their properties, however the nature of their origin is fundamentally different and is determined by the difference of algebraic properties of spaces, where they occur, with respect to the properties of the linear space.

The subject of subsequent discussion will cover the comparative estimation characteristics of signal parameters against the background of interferences (noises) in the linear space and *K*-space of signals. *K*-spaces are well-known for quite a long time and are properly investigated in the available algebraic literature [10, 11]. The use of just *K*-space as a signal space in the subsequent analysis is explained by the fact that as of today it is the only known algebraic structure, where along with the axiomatics of algebraic lattice axioms of the linear space are also satisfied. *K*-space is defined as a linear distributive lattice $L(+,\vee,\wedge)$ over the ring of scalars, where axioms of algebraic distributive lattice $L(\vee, \wedge)$ are satisfied with operations of the upper and lower bounds, namely: $a \vee b = \sup_L\{a, b\}$, $a \wedge b = \inf_L\{a, b\}$ and axioms of the linear space L with addition operation $a + b$ of the additive commutative group $L(+)$ [10, 11]. In this case, elements a, b of the *K*-space can be both the elements of the n-dimensional vector space ($a = [a_1, a_2, \ldots, a_n]$, $b = [b_1, b_2, \ldots, b_n]$) and the function (determinate or random) defined on a certain set T ($a = a(t)$, $b = b(t)$, $t \in T$).

Let us perform a comparative analysis of quality characteristics of estimating the unknown nonrandom parameter $\lambda \geq 0$ for the models of direct measurement in the linear space and *K*-space:

$$\text{– in the linear space } X_i = \lambda + N \tag{1a}$$

$$\text{– in the } K\text{-space } X_i = \lambda \vee N_i, \tag{1b}$$