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Lapunov's Functions Usage in Radio Circuits' Theory

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Abstract—When analyzing linear differential equations' stability using Lapunov's second method, Lapunov's functions are usually chosen in quadratic form. In radio circuits' theory Lapunov's second method by force of existing traditions is applied in rarer cases than for example in mechanics or astronomy. In this article several modifications of the Lapunov's function are suggested on parametric circuit example. Method which is used here applies to linear radio circuits of general type.

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The second Lapunov's stability method has gained wide usage in applied disciplines during last fifty years. The advantage of this method lies in the fact that there is no need to find primitive integral, even qualitative analysis of solutions is not necessary. Within this method special solution function, called Lapunov's function, must be selected then on its base sufficient stability conditions of given solution can be gained. If Lapunov's function is available it does not usually cause difficulties to obtain sufficient stability conditions. If a linear differential equation is analyzed stability theory conceptual apparatus is radically reduced, but the solution of specific stability problems is still difficult. It is proved that in case when at least one solution of a linear equation is stable (unstable) then all its solutions are stable (unstable). That is why trivial (identically equal to the zero) solution of linear equations or linear systems of equations are usually tested for stability. All the more if the linear equation's solution is stable in the neighborhood of some point then all solutions are stable in all existence regions. In linear systems there are no "small" or "large" stability notions which are widely used in automatic control theory. In this case the stability notion considerably extends and refers not to the solution, but to the equation (or to the system of equations which is often considered to be a vector equation). While solving the mentioned problems the main difficulty is to build Lapunov's function, and the second Lapunov's method does not give recommendations on this. More than one hundred years have passed since the second Lapunov's method was formulated. During this time a lot of stability problems were solved by means of this method. Quite various Lapunov's functions were chosen but in linear case they were usually built with quadratic forms. Such choice validation is hard to find in literature. We did not succeed in finding other forms of Lapunov's functions while analyzing linear equations.

At the same time we'll notice, that in electrophysics and radiophysics Lapunov's stability theory does not take hold, unlike mechanics and astronomy. It is rather strange just because coryphaeus of stability theory such as A. M. Lapunov [1], N. G. Chetaev [2], G. N. Duboshin [3], B. P. Demidovitch [4], C. P. Persidskij [5], I. G. Malkin [6], D. P. Merkin [7] were either mathematicians or mechanical engineers. It is difficult to find equivalent specialist in electrical systems' field among them. These vicissitudes of history undoubtedly must be corrected in our time, when systems of radio communications, telephony, automatics and computer engineering have quite evident complication trend.

The given problem we'll study on the example of parametric circuit. One of its equivalent circuits is shown in Fig. 1. As opposed to commonly used circuits here capacity C losses G and inductance L losses R are introduced separately, what conforms to the physics of processes proceeding in the circuit.

Let's consider, that circuit's parameters C, G, L, R vary with time complying any continuous law, staying always positive and do not depend on flowing currents. It is circuit's linearity criterion. In the given case we are interested in a free process. It is well known, that if the problem defined in such a general way the free process can be not decaying (such as in case of a common circuit) but infinitely increasing. Such increasing is often extremely unwanted in a real system that is why reasonably grounded guaranties of such phenomena inadmissibility are sometimes needed. These guaranties can be obtained on the basis of the second Lapunov's method. Let's produce stability proof taking into consideration limitations for circuit's parameters time variation laws shown below.