Radioelectronics and Communications Systems
Vol. 49, No. 1, pp. 46–50, 2006

Izvestiya VUZ. Radioelektronika
Vol. 49, No. 1, pp. 65–72, 2006
UDC 519.7, 621.372

AN APPROACH TO ORDERING OF SYSTEMS OF THE WALSH DISCRETE FUNCTIONS

I. I. Ismagilov

Kazan', Russia

A new approach is suggested to ordering of the Walsh discrete functions based on the properties of their finite-difference representations. The new relationships are used for generating a permutable sequence to construct the difference-ordered system of Walsh functions based on the Walsh-Paley system. Generalization of the Kronecker product of matrices permits to develop a technique of synthesis of the transformation matrix for another version of the difference-ordered system of Walsh functions.

The discrete orthogonal transforms (DOT) have found wide application in treatment of many applied problems related to processing of signals and images by digital methods. By the present time a large number of DOT, as well as general approaches to their generation and investigation, are developed [1–4]. Some of them are very effective in the problems of signal digital processing by spectral methods, and extend its algorithmic arsenal.

In the majority of works devoted to development of the DOT apparatus, primary attention has been given to synthesis of new transforms or modification of the known ones. A much less interest is observed in introduction and study of properties of new orderings of the existing systems of basic functions. It is known, however, that efficiency of treatment of some problems in concern with digital processing of signals depends on ordering of functions in the system. A system composed of N basic vectors can be ordered by $N!$ ways. In practice, however, the known discrete systems use but a small number of fixed techniques of ordering of basic vectors.

A particular place in the totality of practically important DOT is occupied by transformations in various orderings of systems of Walsh discrete functions (WDF). The most well-known orderings of WDF used in signal processing in some system are as follows [1, 4]: sequential ordering (Walsh-Kaczmarz); diadic ordering (Walsh-Paley); and the ordering in conformity with arrangement of rows in Hadamard's matrix (Walsh-Hadamard).

These orderings of the Walsh discrete system of dimensionality $N = 2^n$ are related to each other by the matrices

$$\mathbf{PAL}_N = S_N^{\mathrm{inv}} \mathbf{HAD}_N, \quad \mathbf{WAL}_N = S_N^{\mathrm{iG}} \mathbf{PAL}_N,$$

where $\mathbf{HAD}_N$, $\mathbf{PAL}_N$, and $\mathbf{WAL}_N$ are the transform matrices in the discrete bases after Walsh-Hadamard, Walsh-Paley, and Walsh-Kaczmarz; $\mathbf{S}_N^{\mathrm{inv}}$ is the matrix of binary-inverse permutations; $\mathbf{S}_N^{\mathrm{iG}}$ is the permutation matrix corresponding to the Gray reverse binary code. The techniques of generation of the permutable matrices are given in [2, 4].

Several orderings of WDF systems, leading to symmetric matrices, are considered in [5]. In that work we can find description of fundamental and peripheral symmetric Walsh systems, obtainable by a certain permutation of basic functions in the reference (root) Walsh-Paley matrix. Here the permutable sequences can be determined by using various types of Gray codes and of their generalizations, which can be supplemented by the operation of inverse permutation. Particular interest in symmetric Walsh systems stems from the fact that they are most convenient for spectral analysis. Yet

REFERENCES

1. H. Ahmed and K. R. Rao, Orthogonal Transformations in Digital Signal Processing [Russian translation], Svyaz', Moscow, 1980.

2. E. Ye. Dagman and G. A. Kukharev, Fast Discrete Orthogonal Transforms [in Russian], Nauka, Novosibirsk, 1983.

3. A. M. Trakhtman and V. A. Trakhtman, Foundations of the Theory of Discrete Signals on Finite Intervals [in Russian], Sov. Radio, Moscow, 1975.

4. R. Kh. Sadykhov, P. M. Chegolin, and V. P. Shmerko, Methods and Devices for Signal Processing in a Discrete Basis [in Russian], Nauka i Tekhnika, Minsk, 1987.

5. A. Ya. Beletskii, The Ordered Gray Codes, Uspekhi Sovremennoy Radioelektroniki, No. 11, pp. 3–22, 2003.

6. I. I. Ismagilov, Discrete Transformations in the Bases of Walsh-Like Functions — Theory and Applications in Signal Digital Processing [in Russian], Otechestvo, Kazan', 2003.

7. Yu. M. Smirnov (editor), Design of Special-Purpose Information-and-Computation Systems [in Russian], Vysshaya Shkola, Moscow, 1984.

8 July 2004