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THE HYPERRANDOM FUNCTIONS AND THEIR DESCRIPTION

I. I. Gorban’

Ukrainian Research and Training Center of Standardization, Certification, and Quality Issues, Kiev, Ukraine

The paper presents the essentials of a new mathematical apparatus for describing a special class of indeterminate functions, which cannot be characterized with the probabilistic measure. These functions are termed hyperrandom. For their description some special characteristics are suggested. A hypothesis is put forward stating that all actual events, usually considered as random, are in essence hyperrandom.

Many physical phenomena (electric, acoustic, mechanical etc.) are of indeterminate (indefinite) nature. To describe such phenomena, various probabilistic-stochastic models are often used. However, the possibilities for correct representation of indeterminate phenomena with the aid of random events, random quantities, and random functions are limited. Serious problems arise in the case when, because of changing conditions of observation, we cannot establish some or other statistical rules, even at a large amount of experimental data. Then the known probabilistic and statistical techniques of description of these phenomena turn out to be ineffective. In this context, several works [1, 2] have been devoted to development of principles of description as applied to phenomena watched in nonstationary conditions. Such phenomena were called hyperrandom.

The random and hyperrandom phenomena belong to the same class of indeterminate situations. However, the fundamental difference between them is that the hyperrandom phenomena [1, 2], as distinct from random ones, cannot be characterized with a probabilistic measure [3–5]. In [2] we considered in detail two classes of hyperrandom phenomena — hyperrandom events and hyperrandom quantities. For their description we proposed a mathematical apparatus similar to the probabilistic one — based on non-probabilistic half-measures set on Borel’s field.

The purpose of this paper is to extend the approaches developed in [1, 2] to more complex hyperrandom phenomena — hyperrandom functions — and to show the opportunities of applying the new mathematical apparatus to practical problems.

1. The scalar hyperrandom functions. By the hyperrandom function $X(t)$ is meant a numerical function of an independent argument t, where the function value, at any fixed value of $t \in T$ (where T is the domain of definition of the argument) represents a hyperrandom quantity [2], which is termed “section”. The set of all sections of the hyperrandom function comprises the space of states S (the phase space).

By the ith realization of the hyperrandom function $X(t)$ (sample function) under the condition g is meant a determinate function $x_i(t/g)$, which, for each $i \in I$ (each sampling) and condition g, puts one of the values $x \in S$ into correspondence with each $t \in T$.

The hyperrandom function has the properties of a hyperrandom quantity and of a determinate function. At a fixed value of the argument it turns into a hyperrandom quantity, while at a fixed sampling and fixed condition — into a determinate function. The totality of realizations I may be limited, countable, or uncountable.

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