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THE EQUATIONS OF ELECTRODYNAMICS IN A SPACE WITH ITS METRIC DEFINED THROUGH ROTATION

B. M. Petrov

Taganrog State Radio-Engineering University, Russia

The known covariant Maxwell equations are applied to description of excitation of electromagnetic field by electric charges and currents. The covariant Maxwell equations are formulated for the case of excitation of electromagnetic field by external magnetic charges and currents, supplemented with so-called material equations. The components of the electromagnetic field are identified with the tensor components. Relationships are established between the field intensity and the induction, when passing from Cartesian system to the rotational one and vise versa. Formulation of boundary conditions is presented, and the inhomogeneous wave equations are formulated.

Since the rotational sources of electromagnetic field have found wide application in the antenna and measuring devices [1], and taking into account that the electromagnetic processes occur on the Earth, which is rotating around its axis and moving in the space, and the propagating EM field undergoes the impact of centrifugal and Coriolis forces, our previous works [2–4] were devoted to formulation of some conceptions of applied electrodynamics of the rotating systems of reference in the spherical coordinate system. However, in order to obtain information about rotation frequency and motion speed of radar objects, we often have to treat the electrodynamic problems in the cylindrical coordinate system. Partial solutions to these problems have been published in [5, 6] and other works of the author. In the present paper we set out primary relationships of electrodynamics, which are necessary for solving, in the cylindrical coordinate system, the general problem of excitation of the space whose metric is characterized by rotation.

Introduce in an unlimited space, filled with a homogeneous isotropic linear medium with its permittivity ε and permeability μ, an inertial (Cartesian) system of reference $K'(x', y', z', \mathrm{i}ct) = = K'(r', \mathrm{j}, z', \mathrm{i}ct) = K'(x^{j'})$, where “i” is the imaginary unit, c is the light speed, t is the time, $x^{j'} = (x^{1'}, x^{2'}, x^{3'}, x^{0'})$, $x^{\alpha'} = (r', \varphi', z')$ are cylindrical coordinates. The observation point in this system is denoted as $P'(x^{\alpha'}, \mathrm{i}ct)$. In some part of the space, denoted as the domain V and rotating about P' with a constant angular speed Ω, the external electric and magnetic charges, and currents of the frequency ω_0, measured during the time t, are set. Let us align the origin of the system of reference K' with the center of rotation, and direct the z'-axis along the axis of rotation. Introduce a rotating rigid system of reference $K(x, y, z, t) = K(r, \varphi, z, t) = K(x^j)$, in which the domain V is resting. Denote by $P(x, y, z, t) = P'(x^j)$ the field observation point resting in K. If $x^\alpha = (r, \varphi, z)$ are the cylindrical coordinates then

$$r' = r, \quad \varphi' = \varphi + \Omega t, \quad z' = z. \tag{1}$$

We have to find the vectors of EM field, which satisfy the electrodynamics equations in the systems K' and K, and the radiation condition in the system K.

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