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PARAMETRIC ESTIMATES OF HIGHER-ORDER SPECTRA OF NON-GAUSSIAN STATISTICALLY RELATED PROCESSES

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The eigenfunctions and eigenvectors of the operators of autoregression and of sliding mean are defined in the time and frequency representation. Transformation of higher-order spectra of the non-Gaussian white noise by systems, described by linear prediction models, is analyzed. Expressions are derived for parametric estimation of higher-order spectra of non-Gaussian processes.

A non-Gaussian random process can be fully described by an infinite set of moment or cumulant functions of the second and higher orders. The use, for characterization of non-Gaussian processes, of the second-order estimates of power spectral density (PSD), obtained by the methods of correlograms or periodograms, is incomplete. A non-Gaussian random process can be fully characterized only by the totality of spectra of all orders.

The investigation of higher-order spectra of non-Gaussian processes has not received wide acceptance yet. At the same time, treatment of many applied problems requires statistical estimation of higher-order spectra [1]. For analysis of non-Gaussian processes the third-order periodograms were suggested in [2]. They were obtained as the product of three finite Fourier transforms of non-Gaussian random processes. Another study of cumulant spectra of higher orders was performed in [3, 4]. The Fourier analysis of the moment functions of time series was carried out in some other works [5]. Particularly, Tukey and other authors proposed to name the third-order spectrum as bi-spectrum, while the fourth-order spectrum was called tri-spectrum [2].

In many cases, application of parametric estimation of spectra has advantages over the correlogram or periodogram methods of PSD estimations. The parametric estimates of PSD are calculated from parameters of linear prediction models, obtained by the correlation function. The linear prediction models can be deduced from the moment functions describing the non-Gaussian properties of processes [6]. Such models are called generalized models of linear prediction.

The purpose of this paper is development of the theory of parametric estimation of higher-order spectra based on the models of generalized autoregression (GAR), generalized sliding mean (GSM), and generalized autoregression – sliding mean (GARSM). Below are given the formulas for parametric estimation of higher-order spectra derived from generalized models of linear prediction of an arbitrary rank. For their derivation we used properties of operator equations describing transformations of the moment functions and higher-order spectra of the non-Gaussian white noise in linear systems characterized with linear prediction models.

The operator analysis of discrete linear systems. The expressions for parametric spectral estimation will be obtained based on the operator analysis of discrete linear systems. Although this approach is not widely used in the analysis of linear systems, it permits to pass easily from the equations of time representation to the frequency representation. Using the discrete frequency representation of the operators of autoregression and of sliding mean, we can obtain expressions for parametric assessment of the second and higher-order spectra. In these expressions the normalization coefficients are omitted, because they have been canceled in the equations even in the case of the normalized spectra.

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