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EXACT SOLUTION TO THE TRANSCENDENTAL EQUATION IN THE WAVEGUIDE METHOD OF INVESTIGATION OF DIELECTRICS

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The analytical solution is obtained for a transcendental equation related to measurement of complex permittivity of solid and liquid substances when the measurement is performed by the waveguide method in the microwave range.

The waveguide method has found wide application in investigation of complex permittivity of solid and liquid substances in the microwave range. The method is based on simultaneous measurement of the traveling wave ratio K_{TW} and determination of distance x_m from the surface of the substance under test to the first node of the standing wave [1]. When processing the measurement results, evaluation of the wave propagation constant γ and of the dielectric constant ε involves solving the following transcendental equation

$$\frac{\tanh \gamma d}{\gamma d} = -\mathrm{i}\frac{\lambda}{2\pi d}\frac{K_{\mathrm{TW}} - \mathrm{i}\tan\dfrac{2\pi x_m}{\lambda}}{1 - \mathrm{i}K_{\mathrm{TW}}\tan\dfrac{2\pi x_m}{\lambda}} \tag{1}$$

where d is thickness of the substance under test, and λ is the wavelength in the waveguide.

As noted in [1], the main difficulty of this method arises from impossibility of analytical treatment of equation (1), and from ambiguity because of periodicity of the comprising functions. Up to now, the equation was resolvable only either graphically or numerically — by the method of successive approximations.

Nevertheless, equation (1) can be resolved analytically: either by the Sievert-Burniston method [2] based on the theory of singular integral equations, or by the Luck-Stevens method [3] — with the use of a known Cauchy theorem permitting to determine the integral along the closed contour surrounding a simple pole on the complex plane. However, solutions yielded by these methods represent rather awkward and hardly calculated expressions containing complex-valued integrals, which cannot be taken analytically.

In this paper we suggest a simple and exact analytical solution to equation (1) in terms of γ. We managed to obtain this solution with the aid of new transcendental functions representing a generalization of the Lambert W-function: W_t- and W_c-functions, whose mathematical properties were considered in detail in [4]. We are reminded that the W_t-function is, by definition, the inverse of ($z \tan z$), while the W_c-function is the inverse of ($z \cot z$).

The roots to equation (1) are as follows:

$$\gamma_n = -\frac{\mathrm{i}}{d}W_c^{(n)}\left(\mathrm{i}\frac{2\pi d}{\lambda}\frac{1 - \mathrm{i}K_{\mathrm{TW}}\tan\dfrac{2\pi x_m}{\lambda}}{K_{\mathrm{TW}} - \mathrm{i}\tan\dfrac{2\pi x_m}{\lambda}}\right),\ n = \pm 1, \pm 2, \ldots, \tag{2}$$

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