

## THE NEW DIGITAL LOW-PASS FILTERS

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**Two new low-pass filters with a finite pulse response are suggested. Their main advantages are a rather narrow passband and low level of side lobes of the amplitude-frequency response. To make it more readable, the pulse responses of the filters are represented in table form.**

In addition to low-pass filters (LPF) suggested in [1–4], this paper describes another two LPF with improved amplitude-frequency response (AFR). One of possible applications of the new filters consists in using them as an integral part of the filters designed for approximation of an ideal LPF and of an ideal differentiating filter with prescribed cutoff frequency. Similar to the filters described in [1–4], the new filters use expansion into the Fourier series of polynomial trigonometric kernels of Jackson’s type.

By Jackson’s type kernels are meant the finite trigonometric polynomials  $J_{l,n}(\mu)$  defined for any positive integers  $l$  and  $n$  by the relation  $J_{l,n}(\mu) = C_{l,n} [\sin(n\mu/2) / \sin(\mu/2)]^{2l}$ , where  $C_{l,n}$  is a normalizing coefficient introduced to meet the equality  $\int J_{l,n}(\mu) d\mu = 2\pi$ . Here (and in what follows) the integral without indication of its limits denotes integration over the segment  $\Pi = [-\pi, \pi]$ . For  $l = \overline{1, 5}$  the normalizing coefficients  $C_{l,n}$  have the form

$$\frac{1}{n}, \frac{3}{2n^3 + n}, \frac{20}{11n^5 + 5n^3 + 4n}, \frac{315}{151n^7 + 70n^5 + 49n^3 + 45n}$$

and, respectively,

$$\frac{36288}{15619n^9 + 7350n^7 + 5187n^5 + 4100n^3 + 4032n}.$$

A more thorough information on Jackson’s type kernels is contained in a special monograph [5, chapter II, section 3]. The Fourier expansions of the kernels  $J_{l,n}(\mu)$  for  $l = 2, 3, 4$ , and  $5$  are used in [1–4] for LPF construction. Particularly, in [2, 3] we report exact values of pulse responses for the filters corresponding to Fourier expansions of the kernels  $J_{l,n}(\mu)$  for  $l = 3, 4$ , and  $n = 2^m$ , where  $m = 1, 2, 3$ , and  $4$ . In the present work we give pulse responses for two new LPF. The respective values of the two-dimensional parameter  $(l, m)$  equal  $(3, 5)$  and  $(4, 5)$ . In conformity with the classification introduced in [2, 3], we call them filters No. 5 for  $l = 3$  and, respectively, 4.

The pulse responses  $h_l(t)$  of the suggested filters are defined by relations  $h_l(t) = 2^{-10l} g_l(t)$ ,  $t = \overline{0, 31l}$ , and  $h_l(-t) = h_l(t)$ , and all the nonzero values of the functions  $g_l(t)$  are tabulated in Table 1. The amplitude-frequency responses  $H_l(\omega) = \sum_t e^{-it\omega} h_l(t)$ ,  $\omega \in \Pi = [-\pi, \pi]$  corresponding to the functions  $h_l(t)$ , are described as

$$H_l(\omega) = [\sin 16\omega / 32 \sin(\omega/2)]^{2l}. \quad (1)$$

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