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SYNTHESIS OF A SHAPING FILTER FOR A MARKOV'S CHAIN

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The paper considers a technique for setting the sampled values of a discrete, in time and state, Markovian process based on a priori known probabilistic-in-time mechanism of changing of states. A block diagram of the device realizing this technique is suggested supplemented with an example to explain the principle of operation of the device.

The problem of design of shaping filters assumes a new significance when we deal with extending the effective methods of the stochastic optimal control to the processes which are discrete in state and time, such as Markov's chains (MC). The methods of the theory of state variables, used for describing continuous random sequences, are inconsistent with description of discrete-in-state and discrete-in-time processes because of difficulties in interpretation of the Focker-Planc-Kolmogorov equation for the case under consideration.

The classical representation of Markov's chains [1] is confined to the description of probability-time mechanism of state variation, and gives no way to description of the dynamics of the change of sampled states of MC in the form of adequate state equations. The examples of description of finite-dimensional Markovian sequences are given in [1, 2]. In [1, 2, 3], for a Markov's chain with a finite number of states $X = \{x_1,..., x_i,..., x_m\}$, set by the vector of initial state $\vec{P}(0) = \{p_i(0)\}$, the matrix of single-step transition probabilities $P_{ij}(k/(k–1))$, and by the period of change of the chain state $t_k – t_{k–1} = T$, we can find a notation of the state equation. For this purpose we introduce special indicators of the state of the process $x(k)$ to be modeled:

$$\theta_i(k) = \begin{cases} 1, & \text{at } x(k) = x_i; \\ 0, & \text{in other cases.} \end{cases}$$

In this event the state equation for the process, i.e., $x(k) = C^T \vec{\theta}(k)$, where $C^T = \{x_i\}$ is the row vector of possible states of the process $x(k)$ for which the condition $\{x_1 < x_2 \ldots < x_i \ldots < x_n\}$ is met; n is the state ordinal number which can be written as $\vec{\theta}(k) = P_{ij}^T(k/(k-1))\vec{\theta}(k-1) + \Delta\vec{\theta}(k)$, where $\Delta\vec{\theta}(k)$ is the vector of sequences representing the stepwise martingales compensating the fractional values of the first summand.

However, the authors of the cited work do not disclose the procedure of computation of the vector $\Delta\vec{\theta}(k)$, necessary for providing the preset statistical characteristics of the modeled process. In our opinion, a more appropriate description of discrete Markovian sequences is the following difference stochastic state equation:

$$x(k) = C^T \left(P_{ij}^T(k/(k-1))\vec{\theta}(k-1) \right) + G(k)\nu(k), \tag{1}$$

where $\nu(k)$ is the perturbing random discrete sequence; $G(k)$ is the diffusion coefficient permitting to lead the variance $\nu(k)$ to the variance of the process $x(k)$ for the kth step $G(k) = \sqrt{\sigma_x^2(k)/\sigma_\nu^2(k)}$.

REFERENCES

1. R. Sh. Liptser and A. N. Shiryayev, Statistics of Random Processes (Nonlinear Filtering and Relevant Issues) [in Russian], Nauka, Moscow, 1974.
2. A. Segall, IEEE Trans. Automat. Contr., Vol. 22, No. 2, pp. 179–186, 1977.
3. V. M. Terent'yev and I. B. Parashchuk, Theoretical Foundations of Control of Multichannel Radio-Communication Networks [in Russian], VAS, Sent-Petersburg, 1995.
4. V. Ye. Gmurman, Probability Theory and Mathematical Statistics [in Russian], Vysshaya Shkola, Moscow, 2000.

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