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THE HARTLEY TRANSFORM OF SIGNAL CONVOLUTION

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The paper considers the spectral densities of Hartley's convolution of signals for various linear transformations of its components. The analytical relationships derived for spectral densities of the convolutions of real-valued compound signals are extended to the case of non-analytical complex signals. The presented results of Hartley's spectrum analysis for the main types of its components' transformations facilitate the interpretation of digital processing of compound signals.

With advances in the theory and practice of signal digital processing, an increasing bulk of attention has been given to the methods of spectral analysis based on Hartley's transform (*H*-transform). This is due to the real-valued nature of the Hartley spectral *H*-density $X_H(f)$ of a real signal $x(t)$ and due to the fact that the forward and backward transforms are indistinguishable. All this, in combination with fast algorithms of the discrete *H*-transform, makes it possible to raise substantially the effectiveness of digital processing [1, 2]. At the same time, it would be interesting to compare the processing of compound signals (both real and complex ones) in the time and frequency domains in the case of *H*-transform application.

One of the major operations used for processing of two signals $x(t)$ and $y(t)$ in the time domain is the calculation of the convolution integral (convolution for brevity) of the signals:

$$[x(t) * y(t)] = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) \mathrm{d}\tau.$$

To assure the correctness of the spectral analysis problems, we have to consider the unique features of the *H*-density of the convolution of compound signals, and its properties at various transformations of its components.

When using the *H*-transform of signals, the spectral *H*-density $X_H(f)$ of a real signal $x(t)$ has the form [3]

$$X_H(f) = H[x(t)] = XC(f) + XS(f) = A_x(f) \operatorname{cas} \theta_x(f),$$

where $XC(f) = A_x(f) \cos \theta_x(f)$ is the even function $(XC(f) = XC^{\circ}(f) = (X_H(f) + X_H^{\circ}(f))/2)$ of the Fourier cosine-transform of the signal $x(t)$ (i.e., the Fourier transform of the even component $xc(t) = (x(t) + x^{\circ}(t))/2$ of the signal $x(t)$); $XS(f) = A_x(f) \sin \theta_x(f)$ is the odd function $(XS(f) = -XS^{\circ}(f) = (X_H(f) - X_H^{\circ}(f))/2)$ of the sine-transform of the signal $x(t)$ (i.e., the Fourier transform of the odd component $xs(t) = (x(t) - x^{\circ}(t))/2$); $A_x(f) = A_x^{\circ}(f)$ is the even function of the signal AFS (amplitude-frequency spectrum); $\theta_x(f) = -\theta_x^{\circ}(f)$ is the odd function of the signal PFS; and $\operatorname{cas}(t) = \cos(t) + \sin(t)$ is the Hartley function.

From here on, except as otherwise noted, the superscript $[*]^{\circ}$ denotes inversion (mirror symmetry) of the function [*] about the independent variable

$$x^{\circ}(t) = x(-t), XC^{\circ}(f) = XC(-f), XS^{\circ}(f) = XS(-f), X_H^{\circ}(f) = X_H(-f).$$

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