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APPROXIMATION OF AMPLITUDE-FREQUENCY RESPONSES OF MINIMUM-PHASE RECURSIVE DIGITAL LOW-PASS FILTERS

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The paper considers methods and examples of solution to the problem of approximation of AFR of digital recursive LPF with any ratio between the powers of the numerator and denominator of the transfer function polynomials and with a prescribed function of guaranteed attenuation in the rejection band.

The transfer functions of recursive digital filters (RDF) are described by rational fractions with real-valued coefficients [1, 2]

$$H(z) = \left(\sum_{i=0}^{N} b_i z^{-i} \right) \Big/ \left(1 + \sum_{i=1}^{M} a_i z^{-i} \right). \tag{1}$$

Fraction (1) has $N + M + 1$ variable parameters (coefficients b_i and a_i), which can be used for approximation of prescribed frequency responses of filters to be synthesized. The number of delay (memory) elements, in the case of direct realization of transfer function (1), is equal to the largest value among N and M.

In synthesis of low-pass RDF the bilinear transformation $\Lambda = k(1 - z^{-1})/(1 + z^{-1})$ is used, where Λ is the analog normalized complex-valued variable and k is a constant real coefficient. We can also employ some known solutions to the problem of approximation of frequency responses of analog filters [3]. With the use of polynomial transfer functions $K(\Lambda) = h/\upsilon_n(\Lambda)$, where h is a constant real coefficient and $\upsilon_n(\Lambda)$ is the Hurwitz polynomial of power n [4], we obtain the following transfer function of a digital filter:

$$H(z) = b_0 \left(1 + z^{-1} \right)^n \Big/ \left(1 + \sum_{i=1}^{n} a_i z^{-i} \right).$$

Comparison of the above expression with (1) permits us to conclude that the possibility of variation of the numerator coefficients of the digital transfer function remains untapped.

By using tabulated solutions [5, 6] for analog filters with "equiwave" attenuation characteristics in the passband and with "isoextremal" ones in the rejection band, we obtain the following transfer functions:

$$H(z) = \left(b_0 \prod_{i=1}^{n/2} \left(1 + \gamma_i z^{-1} + z^{-2} \right) \right) \Big/ \left(1 + \sum_{i=1}^{n} a_i z^{-i} \right), \text{ at even } n,$$

$$H(z) = \left(b_0 \left(1 + z^{-1} \right) \prod_{i=1}^{(n-1)/2} \left(1 + \gamma_i z^{-1} + z^{-2} \right) \right) \Big/ \left(1 + \sum_{i=1}^{n} a_i z^{-i} \right), \text{ at odd } n.$$

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