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THE FOURIER TRANSFORM ON THE COMPLEX VARIABLE PLANE

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The paper considers the Fourier transform on the plane of a complex variable, for which the ordinary Fourier transform is a special case. The Fourier transform on the complex variable plane makes it possible to do without the Laplace transform (it is shown that the latter is not the general case of the Fourier transform), since it offers the same computational advantages. Some conditions are defined under which the calculation of the Fourier transforms on the plane of the complex variable can be performed with the aid of existing tables of one-sided Laplace transforms.

Practical calculation of the Fourier transform, occupying an important place in engineering, is often accompanied by some difficulties requiring application of the Laplace transform to overcome them. In such situations, it is usually assumed that the Laplace transform is the general case of the Fourier transform. Let us show the incorrectness of this generalization.

In radioelectronics, acoustical engineering, communications, and some allied applications, the Fourier transform is widely used as a basis for efficient methods of analysis and synthesis of linear systems in the frequency domain. In this case, the forward Fourier transform

$$\Im(\omega) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}t \tag{1}$$

of a process $x(t)$, where $\mathrm{j} = \sqrt{-1}$, is of special significance as a basis of the spectral theory of signals.

In practice, however, especially at calculation of the inverse Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Im(\omega)\mathrm{e}^{\mathrm{j}\omega t}\,\mathrm{d}\omega, \tag{2}$$

some technical difficulties may occur — due to requirement of absolute integrability of the process $x(t)$ and because of complications in calculating improper integrals of functions of a real variable.

In order to overcome these difficulties, researchers often turn to the Laplace transform, well known from the operational calculus, primarily to its one-sided version. Historically, the latter found wide application in engineering much earlier than the Fourier transform. The existence of the Laplace transform

$$L(p) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-pt}\,\mathrm{d}p \tag{3}$$

where $p = \alpha + \mathrm{j}\omega$, does not require the absolute integrability of the process $x(t)$.

In the reverse Laplace transform

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