Radioelectronics and Communications Systems
Vol. 47, No. 2, pp. 6–12, 2004

Izvestiya VUZ. Radioelektronika
Vol. 47, No. 2, pp. 9–17, 2004
UDC 621.372.54.372

ANALYSIS OF FREQUENCY-RESPONSE SENSITIVITY OF THE SECOND-ORDER SECTIONS WITH A REDUNDANT NUMBER OF MULTIPLIERS OF RECURSIVE DIGITAL FILTERS

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The paper considers the possibilities for diminishing the sensitivity of frequency responses to variation of components' parameters and the mean-square sensitivity of frequency responses of minimum-phase and nonminimum-phase sections with an excessive number of components. These sections are used for realization of transfer functions of recursive digital filters obtained at the approximation stage of design.

Transfer functions $H(z)$ of recursive digital filters (RDF), realized by cascade connection of the first- and second-order sections, can be represented as a product of the fractions

$$H(z) = h\frac{1+\beta_0 z^{-1}}{1+\delta_0 z^{-1}} \cdot \prod_{i=1}^{m1} \frac{1+a_{1i} z^{-1} + a_{2i} z^{-2}}{1+b_{1i} z^{-1} + b_{2i} z^{-2}} \cdot \prod_{i=1}^{m2} \frac{1+a_{1i} z^{-1} + z^{-2}}{1+b_{1i} z^{-1} + b_{2i} z^{-2}} = hH_0(z) \cdot \prod_{i=1}^{m1} H_{1i}(z) \cdot \prod_{i=1}^{m2} H_{2i}(z). \quad (1)$$

The multiplier $H_0(z)$ is realized by the first-order section (at $\beta_0 = \delta_0 = 0$ such a section is absent in RDF), The multipliers $H_{1i}(z)$ are realized by nonminimum-phase sections of the second order, while $H_{2i}(z)$ — by minimum-phase sections, whose transfer zeroes lie on the unit circumference in the complex z-plane, i.e., such a section has an AFR zero in the RDF rejection band.

At RDF realization, the storage of the signal values, transfer function coefficients, and results of the multiplication and adding operations is performed in registers with a finite word length. Therefore, prior to the storage, we have to round off (quantize) the numbers, which is the major cause of errors in digital filters (DF). The quantization errors in arithmetic operations depend on the algorithm of DF realization, which explains why in literature one hardly could find any general relations for quantitative estimation of such errors.

Quantization of transfer functions' coefficient values modifies the frequency responses (AFR and PFR) of the realized DF as compared to those obtained at the approximation stage [1]. The word length required for realization of a particular DF can be exactly determined, when we calculate the frequency responses and gradually diminish the word length until the requirements to filter's frequency responses cease to be met. More adaptable is the statistical method [2, 3], permitting not only to determine the necessary word length for a particular network but also to compare various DF networks in this criterion without resort to ultimate calculations. The essence of the statistical method is as follows. By c_i is meant any of the coefficients of transfer function (1) realized as $\pm d \cdot 2^{\nu}$, where d is the normalized mantissa, and ν is the number of digits in the computer word. At small deviations of the parameters c_i from their nominal values the deviation of attenuation $a(x) = -\ln|H(jx)|$ and of APR $\Theta(x) = \arg H(jx)$ are, respectively,

$$\Delta a(x) = \sum_{i=1}^{N} \frac{\partial a}{\partial c_i} \Delta c_i = \sum_{i=1}^{N} S_i E_i, \ \Delta\Theta(x) = \sum_{i=1}^{N} \frac{\partial \Theta}{\partial c_i} \Delta c_i = \sum_{i=1}^{N} R_i E_i \quad (2)$$

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30 April 2002