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REX — THE ORTOGONAL EXPONENTIAL TRANSFORM

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A new orthogonal REX transform is suggested, whose real-valued kernel is based on piecewise exponential functions differentiable on continuous segments. The related transform introduced together with the initial one is called CoREX. Both transforms may undergo modifications to adapt them to signals taken for analysis.

The orthogonal transforms with real kernel are finding increasing use in radio-engineering. A large diversity of such transforms (cosine [1], sine, Hartley's, oblique [2], Walsh [3], various Wavelet-transforms [4]) differ from one another by their width of the transform "spectrum" for one and the same signal (class of signals). Particularly, when synthesizing voice signals, the maximum compression (the least number of transforms without additional compressing coding) is attained if we use the cosine transformation [1]. For description of signals in communication engineering, the Walsh transform is widely used [3]. When we deal with processing of images, technical diagnostics, and pattern recognition, some modifications of the Wavelett-transform have found wide application [4]. In this connection, two lines of investigations are of importance: application of orthogonal transformations already known to the classes of problems where these transformations are optimal, and R&D aimed at new types of orthogonal transformations.

The latter include the new transform called REX (Real Exponential). As distinguished from the known orthogonal transforms with a real kernel, the orthogonal functions of the new transform are differentiable at least at some segments of the time interval $t \in [0, 1]$. Moreover, after differentiation the functions on these segments remain unchanged. Generation of REX functions of this family is carried out by setting the maximum power of the exponential on the interval [0, 1]. For example, if we set the maximal power $e^{1/2}$, the transform Rex$(0, t) = 1$ while the transform Rex$(1, t) = e^t$ on the interval [0, 1/2], and $-e^{\theta}$ on the interval $t \in [1/2, 1]$, and $\theta = t - 1/2$. Similarly, Rex$(2, t)$ is produced from Rex $(1, t)$ by the shift left by an interval $\Delta T = 1/4$. The ensuing group of functions Rex $(3, t)$,..., Rex $(6, t)$ is formed with the aid of the generating Rex $(3, t)$, which now contains 4 alternating exponentials with maximal powers $e^{1/4}$. Particularly, Rex$(4, t)$ is produced from Rex $(3, t)$ by a shift left by $\Delta T = 1/8$, while other functions (Rex $(5, t)$ and Rex $(6, t)$) acquire additional jumps on the function Rex $(4, t)$, where the function signs alter in accordance with the same rule as the Walsh functions wal$(5, t)$ and wal$(6, t)$ if being reordered by the Karchmarzh rule. Thus, the functions Rex(n, t) form intervals with alternating signs, which are coincident with the alternating intervals of the functions wal(n, t). However, on the intervals where the function Rex(n, t) has no jumps, it varies exponentially.

Table 1 contains the values of the function Rex(n, t) corresponding to the discrete REX-transform for the first 8 functions (the transform matrix has the order $N = 2^3 = 8$, and the maximal power of the exponential is $e^{1/2}$).

The orthogonality of the functions listed in Table 1 can be easily confirmed if we perform multiplication of the rows at any quantization step and any order of the matrix $\overline{\overline{\overline{\text{Rex}}}}$. The maximal power of the exponential may be set in arbitrary manner. As a result, we can obtain an infinite set of the functions Rex(n, t). After normalization, as the maximum power of

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