THE CORRESPONDENCE BETWEEN TWO-DIMENSIONAL FOURIER AND HARTLEY TRANSFORMS

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The author considers the relation between two-dimensional Fourier and Hartley transforms of two-dimensional real- and complex-valued signals based on analysis of symmetric and antisymmetric components of these signals.

The methods of spectral analysis based on a pair of two-dimensional Fourier $F$-transformations

$$Z_F (X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z(x,y) \exp(-j(2\pi x X + 2\pi Y y)) \, dx \, dy,$$

$$z(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_F (X,Y) \exp(j(2\pi x X + 2\pi Y y)) \, dX \, dY,$$

have gained wide-spread acceptance in theoretical and applied investigations of two-dimensional signals. Here “the signal” $z(x,y)$ is a real-valued function of variables $x$ and $y$, while $Z_F (X,Y)$ is the complex-valued Fourier spectral density ($F$-density) considered as a function of “frequencies” $X$ and $Y$ [1].

In practice, when we deal with various transformations of the $F$-density $Z_F (X,Y)$ of a real-valued signal $z(x,y)$ (weighted smoothing, rotation, displacement of origin, filtration etc.), the inverse $F$-transformation of the already transformed function $\tilde{Z}_F (X,Y)$ often leads to a complex signal $\tilde{z}(t)$ whose components’ interpretation and comparison with the signal $z(x,y)$ present some difficulties. This situation complicates substantially the comparison of the results of analysis of nonstationary and nonlinear systems used for processing of sophisticated two-dimensional signals [1].

At the present time, the preference is given to the spectral analysis methods whose main part includes the many-dimensional Hartley transformation. It stems from the real-valued nature of the many-dimensional Hartley density ($H$-density), impossibility to distinguish the direct and inverse many-dimensional Hartley transform, and from the real-valued nature of the signal $\tilde{z}(t)$ corresponding to an arbitrary transformation of the spectral $H$-density of the signal $z(x,y)$ [2, 3].

Two types of two-dimensional $H$-transformation are known. The first type is related with the additive argument of the kernel of a pair of $H$-transforms such as

$$Z_{H+H} (X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z(x,y) \cos(2\pi(xX + yY)) \, dx \, dy,$$

$$z(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_{H+H} (X,Y) \cos(2\pi(xX + yY)) \, dX \, dY.$$

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REFERENCES


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