Radioelectronics and Communications Systems
Vol. 46, No. 1, pp. 5–10, 2003

Izvestiya VUZ. Radioelektronika
Vol. 46, No. 1, pp. 8–15, 2003
UDC 621.397, 621.37

THE CORRESPONDENCE BETWEEN TWO-DIMENSIONAL FOURIER AND HARTLEY TRANSFORMS

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The author considers the relation between two-dimensional Fourier and Hartley transforms of two-dimensional real- and complex-valued signals based on analysis of symmetric and antisymmetric components of these signals.

The methods of spectral analysis based on a pair of two-dimensional Fourier F-transformations

$$Z_F(X,Y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(x,y)\exp(-j(2\pi xX+2\pi yY))\mathrm{d}x\mathrm{d}y,$$

$$z(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Z_F(X,Y)\exp(j(2\pi xX+2\pi yY))\mathrm{d}X\mathrm{d}Y,$$

have gained wide-spread acceptance in theoretical and applied investigations of two-dimensional signals. Here “the signal” $z(x, y)$ is a real-valued function of variables x and y, while $Z_F(X,Y)$ is the complex-valued Fourier spectral density (F-density) considered as a function of “frequencies” X and Y [1].

In practice, when we deal with various transformations of the F-density $Z_F(X,Y)$ of a real-valued signal $z(x, y)$ (weighted smoothing, rotation, displacement of origin, filtration etc.), the inverse F-transformation of the already transformed function $\widetilde{Z}_F(X,Y)$ often leads to a complex signal $\widetilde{z}(t)$ whose components’ interpretation and comparison with the signal $z(x, y)$ present some difficulties. This situation complicates substantially the comparison of the results of analysis of nonstationary and nonlinear systems used for processing of sophisticated two-dimensional signals [1].

At the present time, the preference is given to the spectral analysis methods whose main part includes the many-dimensional Hartley transformation. It stems from the real-valued nature of the many-dimensional Hartley density (H-density), impossibility to distinguish the direct and inverse many-dimensional Hartley transform, and from the real-valued nature of the signal $\widetilde{z}(t)$ corresponding to an arbitrary transformation of the spectral H-density of the signal $z(x, y)$ [2, 3].

Two types of two-dimensional H-transformation are known. The first type is related with the additive argument of the kernel of a pair of H-transforms such as

$$Z_{H+H}(X,Y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(x,y)\,\mathrm{cas}(2\pi(xX+yY))\mathrm{d}x\mathrm{d}y;$$

$$z(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Z_{H+H}(X,Y)\,\mathrm{cas}(2\pi(xX+yY))\mathrm{d}X\mathrm{d}Y.$$

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31 May 2001