RESISTANCE OF A FILM RESISTOR WITH A SLIT OF FINITE WIDTH

V. D. Sadkov and D. E. Orlov

Izvestiya VUZ. Radioelektronika,
Vol. 35, No. 11, pp. 78-80, 1992

UDC 621.396.6-181.5

Film resistors with an infinitesimally thin correcting slit were calculated in [1, 2]. We will take into account the finite width of a slit (Fig. 1) by the method of conformal mapping [3]. It is more convenient to solve the problem with inversion of the boundary conditions - transfer of contacts from sections M_1N_1 and M_6N_5 of the film to sections $M_1M_2M_3$-$M_4M_5M_6$ and N_1N_5 (the resistance of the investigated R_{invs} and inverted R_{invr} resistors are associated by the relation: $\sqrt{R_{invs}R_{invr}} = \rho$, where ρ is the resistivity of the resistive film. We will consider that contact N_1N_5 has potential $U = U_0$, and contact $M_1M_2M_3M_4M_5M_6$ has $U = 0$.

We will carry out conformal mapping of strip NN'M' with slit $M_2M_3M_4M_5$ of region Z (Fig. 1a) onto the upper half-plane ζ (Fig. 1, b) with the indicated correspondence of points by means of the Schwarz formula [3]

$$\omega = (1/\pi i)\int_{-\infty}^{\infty} [U(\xi)/(\xi-\zeta)]\,d\xi = (U_0/\pi i)\ln[(\zeta-1)/(\zeta+1)].$$

The magnitude of the current flowing through section N_1N_5

$$i = (1/\rho)\int_{N_1N_5} (d\omega/dy)\,dx = (4U_0/\rho\pi)\,\mathrm{Arth}\,\gamma.$$

When there is no slit, the current through this section $i_0 = (2U_0/\rho)a/(c+d)$. Then the ratio of the currents m for the inverted and investigated resistors

$$m_{invr} = 1/m_{invs} = i/i_0 = (2/\pi)[(c+d)/a]\,\mathrm{Arth}\,\gamma. \quad (1)$$

To find the parameter γ in (1), we will construct for strip NN'M'M with a slit a mapping function with the use of the Christoffel-Schwarz integral [3]:

$$(\pi/2)z/(c+d) = C\int_0^{\zeta}\{\sqrt{[(\zeta^2-\alpha^2)/(\zeta^2-\beta^2)]}/(1-\zeta^2)\}\,d\zeta, \quad (2)$$

where α, β are mapping parameters; $C = \sqrt{[(1-\beta^2)/(1-\alpha^2)]}$ is a mapping constant. From (2) we obtain:

$$\delta = (\pi/2)b/(c+d) = C\int_0^{\alpha}\{\sqrt{[(\alpha^2-\xi^2)/(\beta^2-\xi^2)]}/(1-\xi^2)\}\,d\xi;$$
$$\kappa = (\pi/2)d/(c+d) = C\int_0^{\beta}\{\sqrt{[(\xi^2-\alpha^2)/(\beta^2-\xi^2)]}/(1-\xi^2)\}\,d\xi. \quad (3)$$

Reducing the elliptic integrals to a normal form, we have [4]

$$\delta = (C/\beta)[K-(1-\alpha)\Pi(-\alpha^2,k)];$$
$$\kappa = (C/\beta)\{(1/C^2)\Pi[(\beta^2-\alpha^2)/(1-\beta^2),k'] - K'\}, \quad (4)$$

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2 March 1992