EFFECTIVE ALGORITHM FOR STATISTICAL DIGITAL FILTERING

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A rigorous mathematical solution of the problem of statistical synthesis of recursive digital filters based on R. L. Stratonovich's theory of conditional Markov processes was given in [1]. However, the distortions specific for microprocessors due to rounding of numbers and limitation of the word length, which lead to substantial estimation errors, were not taken into account here. These errors are caused by loss of positive definiteness and ill condition of the correlation matrices. To overcome the indicated difficulties, the method of taking the square roots of matrices is used successfully in Kalman filters [2]. Algorithms for digital filtering resistant to computational errors related to limitation of the word length of microprocessors on which the digital filter is realized are obtained in the work by means of the given method. To synthesize the algorithms by the proposed method, we will use an a priori equation of the Markov processes being estimated

$$\begin{bmatrix} X_i \\ y_i \end{bmatrix} = \begin{bmatrix} \Phi_{xx} & \vdots & 0 \\ \cdots & \vdots & \cdot \\ H\,\Phi_{xx} & \vdots & 0 \end{bmatrix} \begin{bmatrix} X_{i-1} \\ y_{i-1} \end{bmatrix} + \begin{bmatrix} \Gamma_{xx} & \vdots & 0 \\ \cdots & \vdots & \cdot \\ H\,\Gamma_{xx} & \vdots & 0 \end{bmatrix} \begin{bmatrix} N_x(i-1) \\ 0 \end{bmatrix},$$

where X_i is the Markov vector of signals and noise; y_i is an analog scalar process of a mixture of signals with noise; Φ_{xx}, Γ_{xx}, H are known matrices; N_x is the vector of Gaussian processes with statistical characteristics

$$M\{N_x\} = 0,\; M\{N_x(i)\, N_x^{\mathrm{T}}(j)\} = I\,\delta_{ij};$$

δ_{ij} is the Kronecker delta; I, 0 are unit and zero matrices; T is the transposition operation. The optimal estimate of the vector $[X_i^{\mathrm{T}}, y_i]^{\mathrm{T}}$ is determined according to the rule of a posteriori expected value on the basis of observed digital signals $s_i(r)$, which are formed by an analog-to-digital converter (ADC) at discrete times i = 1, 2, ... The dynamic range of the ADC is divided into quantization regions $\Theta_i(r)$ by quantization thresholds $h_i(r)$ and $h_i(r-1)$, where r is the number of the quantization region. If the input quantity $y_i \in \Theta_i(r)$, then the ADC forms a digital signal $s_i(r)$.

The optimal estimate of vector X_i is determined on the basis of observed digital signals $S_1^i = \{S_1, S_2, ..., S_i\}$ and has the form [1]

$$X_i^* = \Phi_{xx}(i, i-1)\, X_{i-1}^* + K_i\, y_{\mathrm{opt}}[i, s_i(r)]; \tag{1}$$

$$K_i = K_{xx} H^{\mathrm{T}} (H K_{xx} H^{\mathrm{T}})^{-1}. \tag{2}$$

Here K_i is the optimal filter gain; K_{xx} is the correlation matrix of errors of extrapolating the signals and noise

$$K_{xx}(i) = \Phi_{xx} E_{xx}(i-1)\, \Phi_{xx}^{\mathrm{T}} + \Gamma_{xx} \Gamma_{xx}^{\mathrm{T}}, \tag{3}$$

y_{opt} is the optimal quantization levels, determined from the expression

$$y_{\mathrm{opt}} = \frac{\Phi^{(1)}[h(r-1)] - \Phi^{(1)}[h(r)]}{\Phi[h(r)] - \Phi[h(r-1)]},$$

where $\Phi[\cdot]$ and $\Phi^{(1)}[\cdot]$ are the error function and its derivative.

The correlation matrix of the errors of digital filtering of signals and noise

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