# ALGORITHMS FOR DETERMINING THE DISTRIBUTION MOMENTS OF A RANDOM PROCESS USING A CHARACTERISTIC FUNCTION

**Yu. M. Veshkurtsev and S. M. Novikov**

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From [1,2], algorithms for determining the probabilistic characteristics of random processes are well known; these algorithms are constructed on the basis of estimates of the real and imaginary parts of the characteristic function and have the form

$$A(V)=\int_0^T g(T-t)\cos[Vx(t)]\,dt;\qquad B(V)=\int_0^T g(T-t)\sin[Vx(t)]\,dt, \tag{1}$$

where x(t) is the investigated random process; V is the parameter of the characteristic function; g(T - t) is the pulse response of the integrator; A(V), B(V) are respectively the estimates of the real and imaginary parts of the characteristic function. In [1,2] the following formulas are given for determining the initial distribution moments:

$$\begin{gathered} m_\alpha=(2/x_m)\sum_{k=1}^{\infty}B(k\Delta V)\int_0^{x_m}x^\alpha\sin(xk\Delta V)\,dx;\\ m_n=x_m^n/(n+1)+(2/x_m)\sum_{k=1}^{\infty}A(k\Delta V)\int_0^{x_m}x^n\cos(xk\Delta V)\,dx, \end{gathered} \tag{2}$$

where x, $x_m$ are the current and maximum values of the random process x(t); ΔV is the quantization step of the real parameter of the characteristic function; V = kΔV; α = 2r - 1; n = 2r; r is a prime number. In accordance with Eqs. (1), the analyzer measures the estimates of the real (A(V) and imaginary B(V) parts of the characteristic function, while in the computer block the algorithms (2) are used to determine the initial (central) distribution moments of the random process.

The purpose of the present work is to investigate the known (2) and newly derived algorithms for determining the distribution moments of random processes and developing recommendations for their application.

Using the expansion of the characteristic function into a Maclaurin series (see [3]) for various values of the real parameter V, we derive a system of linear equations

$$\begin{cases} A(1\Delta V)+jB(1\Delta V)=1+j1\Delta Vm_1-[(1\Delta V)^2/2!]\,m_2+\ldots+[(j1\Delta V)^r/r!]\,m_r;\\ A(2\Delta V)+jB(2\Delta V)=1+j2\Delta Vm_1-[(2\Delta V^2/2!]\,m_2+\ldots+[(j2\Delta V)^r/r!]\,m_r;\\ \cdots\\ A(k\Delta V)+jB(k\Delta V)=1+jk\Delta Vm_1-[(k\Delta V)^2/2!]\,m_2+\ldots+[(jk\Delta V)^r/r!]\,m_r, \end{cases}$$

where j is the imaginary unit; k = r/2 is the number of equations. The solution of this system of equations for the i-th initial distribution moment $m_i$ has the form

$$m_\alpha=\sum_{k=1}^{\infty}b_\alpha(k\Delta V)\,B(k\Delta V);\qquad m_n=c_n+\sum_{k=1}^{\infty}a_n(k\Delta V)\,A(k\Delta V), \tag{3}$$

where $a_n(k\Delta V)$, $b_\alpha(k\Delta V)$, $c_n$ are coefficients obtained in the solution of the system of linear equations.

An analogous system of equations may be formulated if the expansion of the cumulant function into a Maclaurin series is used along with its relationship to the characteristic function (see [3]). Then the solution of the

## REFERENCES


1. P. V. Mel'nikov, "Concerning the problem of measuring the characteristic function of a stationary ergodic process," Elektrosvyaz, no. 12, pp. 19-25, 1962.
2. Yu. M. Veshkurtsev, "Estimating the error of determining the statistical characteristics of phase fluctuations using an analyzer having an ideal channel for forming the reference wave," Avtometriya, no. 6, pp. 48-53, 1983.
3. B. R. Levin, Theoretical Foundations of Statistical Radioengineering [in Russian], vol. 1, Sov. radio, Moscow, 1969.
4. G. Korn and T. Korn, Handbook on Mathematics of Scientific Workers and Engineers [in Russian], Nauka, Moscow, 1984.
5. V. I. Kinshchak and A. V. Nesterov, "Selection of the quantization frequency in measuring the characteristics of a random process by discrete methods," Izv. VUZ. Radioelektronika [Radioelectronics and Communications Systems], no. 1, pp. 108-110, 1979.


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