FOURIER CONJUGACY OF GREEN'S TENSOR AND OF THE DIFFRACTION FIELD OF A PLANE WAVE ON AN IDEALLY CONDUCTING BODY

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In solving external problems in electrodynamics, it is sometimes necessary to determine the field produced by the diffraction of a plane wave on an ideally conducting body according to the well-known Green tensor (GT) of this body. The inverse problem is likewise of practical interest - i.e., the determination of the GT according to the known field produced by the diffraction of a plane wave on an ideally conducting body. As will be demonstrated below, these two problems are interrelated, while the diffraction field and the GT are Fourier-conjugate relative to each other. In scalar form, this interrelationship has been considered in [1, p. 81] for Green's function of free space.

Let us consider the solution of the first problem for an ideally conducting body which is situated at the origin of a rectangular coordinate system x, y, z. The electric E_p and magnetic H_p fields of the plane wave incident on the body (the time dependence is exp(iωt); ω is the angular frequency; t is time) are written in the form

$$E_p = E_0 \exp[-i(k_x x + k_y y + k_z z)], \quad H_p = H_0 \exp[-i(k_x x + k_y y + k_z z)], \tag{1}$$

where k_x, k_y, k_z are the projections of the wave vector k onto the coordinate axes; E_0, H_0 are unit vectors which are independent of the coordinates and determine the spatial orientation of the electric and magnetic fields, respectively. The fields E_p, H_p satisfy the homogeneous Maxwell equations from which one can find the spatial orientation of one of the vectors E_0, H_0 by stipulating the orientation of the other one.

Let us find the fields E_s, H_s scattered by the body from the inhomogeneous wave equations

$$\nabla^2 E_s + k^2 E_s = -M^e; \qquad \nabla^2 H_s + k^2 H_s = -M^M, \tag{2}$$

where M^e, M^M are auxiliary vectors:

$$\begin{aligned} M^e &= -i\omega\mu j^e + (1/i\omega\varepsilon)\,\mathrm{graddiv}\, j^e - \mathrm{rot}\, j^M; \\ M^M &= -i\omega\varepsilon j^M + (1/i\omega\mu)\,\mathrm{graddiv}\, j^M + \mathrm{rot}\, j^e. \end{aligned} \tag{3}$$

In Eqs. (3) j^e and j^M denote the equivalent electric and magnetic currents determined by the field vectors of the incident wave. Let us determine these currents on some closed surface encompassing the body. As such a surface we shall choose a portion of a sphere having an infinitely large radius, which is bounded by an infinite plane which forms a nonzero acute angle with the incident wavefront (IWF) and is situated relative to the body in that portion of the space from which the plane wave is incident. Zero and right angles between the IWF and the plane of equivalent currents (PEC) are excluded, since at these angles $M^e = 0$, $M^M = 0$ in Eqs. (2). Figure 1 displays the mutual arrangement of the scattering ideally conducting body, the PEC, and the IWF, the z axis of the chosen coordinate system being perpendicular to the PEC and the two planes intersecting at the point z_0. The equivalent currents in Eqs. (3) are stipulated by the relationships

$$j^e = [H_p, n]; \qquad j^M = [n, E_p], \tag{4}$$

where n is the unit vector of the external normal to the PEC. Let us take account of the fact that in the PEC the following relationships (see [2]) are valid: $\mathrm{rot}\, j^e = -i\omega\varepsilon j^M + (1/i\omega\mu)\,\mathrm{graddiv}\, j^M$; $\mathrm{rot}\, j^M = i\omega\mu j^e - (1/i\omega\varepsilon)\,\mathrm{graddiv}\, j^e$

REFERENCES

1. O. N. Litvinenko, Foundations of Radio-Optics [in Russian], Tekhnika, Kiev, 1974.

2. D. I. Voskresenskii and V. M. Maksimov, "Generalization of the Schwarzschild method for the problem of scattering by ideally conducting bodies," Izv. VUZ. Radioelektronika [Radioelectronics and Communications Systems], no. 2, pp. 15-19, 1988.

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