# EXACT DISTRIBUTION OF THE LEVELS OF PERIODIC CORRELATION FUNCTIONS OF COMPLETE AND TRUNCATED BINARY CODES

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The selection of signal systems for M-ary data-transmission channels is frequently based on orthogonalization of the system (see [1]). At the same time, it is expedient in a number of applications to use random binary codes, while a system of M signals may be formed by cyclical shifts of the code sequence formed by a M-digit random-number generator. In the first approximation, the estimate of the correlation properties of such codes has been given in [2], while in [3] an analysis was performed of the energy losses of channel interference immunity for conversion from orthogonal codes to random codes. These losses turned out to be comparatively low (about 1.5 dB for an error probability $p_e = 10^{-3}$).

The Gaussian approximation derived in [2] for the levels of the periodic correlation function (PCF) regrettably yields a very high error on the distribution boundaries; this leads to incorrect estimates of interference immunity in the region of high signal/noise ratios. The present work presents a procedure for determining the exact distribution of PCF levels for complete and truncated binary codes, and the distributions are likewise obtained for the PCF levels in the case of codes having a length $N = 2^p$ ($p = 2$ to 5).

The PCF of the j-th code vector $\alpha_0^j = (a_0^j, a_1^j, \ldots, a_{N-1}^j)$ having the elements $a_i^j \in \{0, 1\}$ of a complete binary code is defined as

$$R_{am}^j = \sum_{i=0}^{N-1} a_i^j \oplus a_{i+m}^j = \sum_{i=0}^{N-1} r_{m,i}^j = K_m^j, \quad m = 0 \ldots N-1, \; j = 1 \ldots 2^N, \tag{1}$$

where $\oplus$ is the symbol representing the sum modulo 2, $i + m \equiv (i + m) \bmod N$ (see [4]).

In other words, $R_{am}$ may be treated as the Hamming distance between the code vector $\alpha_0^j = (a_0^j, a_1^j, \ldots, a_{N-1}^j)$ and its m-th cyclical shift $\alpha_m^j$ or as the weight k of the vector of the sum $\rho_m^j = \alpha_0^j \oplus \alpha_m^j$.

As is well known, the weights of the vectors $(a_0^j, a_1^j, \ldots, a_{N-1}^j)$, $j = 1 \ldots 2^N$ of the complete code are distributed according to a binomial law (see [1]). The distribution of the weights of the vectors $(r_{m,0}^j, r_{m1}^j, \ldots, r_{m,N-1}^j)$ depends on the cyclical shift m between the vectors $\alpha_0^j$ and $\alpha_m^j$. Let us consider the most interesting practical case $N = 2^p$ in which each cyclical shift may contain p bits of information. From Eq. (1) it is not difficult to derive the equations:

$$\begin{gathered} r_{m,0}^j \oplus r_{m,1}^j \oplus \cdots \oplus r_{m,N-1}^j = 0; \quad r_{m,i}^j \oplus r_{m,2+i}^j \oplus \cdots \oplus r_{m,N-2+i}^j = 0, \\ i = 0, 1, m \equiv 0 \bmod 2; \quad r_{m,i}^j \oplus r_{m,4+i}^j \oplus \cdots \oplus r_{m,N-4+i}^j = 0, \; i = 0 \ldots 3, \; m \equiv 0 \bmod 4; \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ r_{m,i}^j \oplus r_{m,N/2+i} = 0, \quad i = 0 \ldots (N/2 - 1), \quad m \equiv 0 \bmod N/2. \end{gathered} \tag{2}$$

Based on this, one can determine the relationship for the weights of the vectors $\rho_m^j = (r_{m,0}^j, r_{m,1}^j, \ldots r_{m,N-1}^j)$:

$$\sum_k C_N^k = 2^{N-1}, \quad k = 0, 2, 4, \ldots, N, \quad m \equiv 0 \bmod 2; \tag{3}$$

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