METHOD OF INTEGRAL REFERENCE CURVES FOR THE ANALYSIS OF STOCHASTIC SYSTEMS

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A numerical-analytic method has been developed for analyzing stochastic systems, which ensures a substantial reduction of computational expenditures compared with traditional approaches for stipulated accuracy characteristics.

The evolutionary Fokker-Planck-Kolmogorov equations (for noninteracting systems having no input sampling) and Stratonovich equations (for interacting systems having input samplings) are used extensively for analyzing nonlinear stochastic systems with resort to the theory of Markovian processes. Since effective methods for the exact integration of the indicated partial differential equations practically do not exist, it follows that the analysis of stochastic systems is, as a rule, restricted to the application of various approximate approaches. The majority of them is associated with the necessity of performing numerical simulation of systems of ordinary differential equations describing the time evolution of the ensemble of parameters characterizing the a priori probability density for the Fokker-Planck-Kolmogorov equation and the a posteriori probability density for the Stratonovich equation. Well known highly accurate methods of numerical integration (for example, the Runge-Kutta fourth-order method) may be used to simulate such equations; however, under these conditions the resources of the digital simulating aggregate are expended irrationally. This is due to the necessity of multiple repetition of the complete program of calculations associated with the plotting of the next integral curve corresponding to the new initial conditions.

The present work considers a numerical-analytic method of integrating evolutionary equations which are used in the analysis of stochastic systems; this method, unlike the traditional approaches, allows a substantial reduction of the computational expenditures.

Let us consider the evolutionary partial differential equation (see [1]) for the M-dimensional Markovian process x(t):

$$\partial p(x, t)/\partial t = L_t^{(M)}\{p(x, t)\}, \quad x \in X \in R^M, \quad t \in T \in R^1, \tag{1}$$

where $L_t^{(M)}$ is the a priori Fokker-Planck-Kolmogorov operator in the analysis of noninteracting systems, and the a posteriori Stratonovich operator in the analysis of interacting systems.

It is well known that the integration of Eq. (1) may be reduced with any predetermined accuracy to the solution of the Cauchy problem for a normal system of ordinary differential equations which is a parametric description of the desired probability density:

$$d\lambda/dt = f(\lambda, t, y(t)), \quad \lambda \in \Lambda \in R^N, \quad t \in T \in R^1, \tag{2}$$

where $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_N]^T$ is an N-dimensional vector of time dependent parameters according to which an approximate solution of Eq. (1) can be uniquely restored; f is a vector of known functions for which the conditions of the existence and uniqueness of the solution of Eq. (2) are satisfied; y(t) = S(t, x) + n(t) is the input sampling (for interacting systems, y(t) = 0).

The traditional analysis of stochastic systems within the framework of Markovian theory is based on the numerical solution of the Cauchy problem (2) in a stipulated domain $\Lambda \times T$ for a large number of different initial conditions. Let us consider a different approach to the solution of the problem given, which allows an approximate analytic solution of Eq. (2) (and consequently of Eq. (1) as well) to be obtained with any predetermined accuracy

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