# JOINT FILTERING OF MIXED MARKOV PROCESSES IN DISCRETE TIME

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An optimal and a quasi-optimal algorithm for filtering mixed Markov processes in discrete time, in which the discrete component is a Markov chain while the continuous component consists of sections of Markov sequences, are synthesized. Using the criterion of minimum a posteriori risk, a Bayes decision rule is obtained for one form of the loss function. The well-known and the quasi-optimal filtering algorithm synthesized here are compared using statistical modeling on a computer.

Mixed Markov processes are widely used to describe stochastic objects with a random structure. In [1,2], the problems of filtering discrete-continuous Markov processes and processes with a random structure in continuous time are solved. The filters synthesized relate to the class of devices with decision feedback.

Processes in discrete time, in which the discrete component is a Markov chain while the continuous component consists of sections of Markov sequences are an important class of Markov processes. One of the methods of synthesizing optimal algorithms for filtering these processes is considered in [3]. The optimal filter synthesized in that paper relates to the class of devices with an increasing memory. The quasi-optimal filtering algorithm is obtained as a result of a Gaussian approximation of the a posteriori probability density of the continuous component of the mixed Markov process.

In this paper, using the theory of conditional Markov processes [1], we synthesize recurrent optimal and quasi-optimal algorithms for filtering mixed Markov processes in discrete time.

To synthesize the algorithm using the proposed method one can use the following equations, which describe a mixed process in discrete time:

$$x(k) = F[x(k-1), A(k)] + G[A(k)]\,w(k); \tag{1}$$

$$y(k) = h[x(k), B(k)] + C[B(k)]\,v(k), \tag{2}$$

where x(k) is an $N_1$-dimensional continuously valued vector, A(k), B(k) are the Markov chains with states $a_j, j=\overline{1, M_1}, b_m, m=\overline{1, M_2}$ and transition matrices $\Pi^1_{ij}(k, k-1), i, j=\overline{1, M_1}, \Pi^2_{nm}(k, k-1), n, m=\overline{1, M_2}$, respectively, F[·] is an $N_1$-dimensional vector-valued function, G[·] is a matrix with dimensions $N_1 \times N_2$; $w(k)$ is an $N_3$-dimensional uncorrelated Gaussian sequence with zero expectation value and correlation matrix Q(k), y(k) is an $N_2$-dimensional measurement vector, h[·] is an $N_3$-dimensional vector-valued function, C[·] is a matrix of dimensions $N_3 \times N_3$; $v(k)$ is an $N_3$-dimensional uncorrelated Gaussian sequence with zero expectation value and correlation matrix R(k).

Following the method proposed in [1], we will denote the joint probability density of the mixed process by $\mathbf{P}(x(k), A(k)=a_j, B(k)=b_m)$. The quantity $\mathbf{P}(x(k), A(k)=a_j, B(k)=b_m)\,dx(k)$ is equal to the probability of simultaneously satisfying the conditions $x(k) \in (x(k), x(k)+dx(k))$, $A(k)=a_j, B(k)=b_m$. The Markov chain B(k) is independent of the processes x(k) and A(k), and

## REFERENCES

1. V. I. Tikhonov and N. K. Kul'man, Nonlinear Filtering and Quasi-Coherent Signal Reception [in Russian], Sovetskoe Radio, Moscow, 1975.

2. I. E. Kazakov and V. M. Artem'ev, Optimization of Dynamic Systems with a Random Structure [in Russian], Nauka, Moscow, 1980.

3. C. A. Ackerson and S. Fuk, "On state estimation in switching environments," IEEE Trans.: V-AC-15, no. 1, pp. 10-17, 1970.

4. V. I. Tikhonov, V. N. Kharisov, and V. A. Smirnov, "Optimal filtering of discrete-continuous processes," Radiotekhnika i Elektronika, no. 7, pp. 1441-1457, 1978.

5. E. Sage and J. Melsa, Theory of Estimation and its Application in Communication and Control [Russian translation], Svyaz, Moscow, 1976.

6. V. G. Repin and G. P. Tartakovskii, Statistical Synthesis in the Case of A Priori Ambiguity and Adaptation of Information Systems [in Russian], Sovetskoe Radio, Moscow, 1977.

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